The value of travel time variability with trip chains, flexible scheduling and correlated travel times

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Abstract

This paper extends the analysis of the value of mean travel time (VMTT) and day-to-day travel time variability (VTTV) from single, isolated trips to daily trip chains, considering the effects of flexibility in activity scheduling and within-day correlation of travel times. Using a multi-stage stochastic programming approach, we show that the VMTT and VTTV on a trip is conditional on the realized travel times on preceding trips, first through the arrival time to the preceding activity and second through the information provided about subsequent travel times. Analytical formulas for the VMTT and VTTV are obtained for two special cases with piecewise constant and linear marginal cost functions, respectively. With flexible scheduling, there is typically a cost associated with a positive correlation of travel times, arising from persistent deviations from typical travel demand or supply on a given day. However, there is also a strict benefit in the dependence since it allows for a more efficient scheduling of later trips.

KEYWORDS: travel time variability, reliability, travel time correlation, trip chains, activity scheduling

1 Introduction

1.1 Background and previous work

In recent years it has been increasingly recognized that there are costs associated with not only the typical (mean) travel time, but also with the unpredictable variation of travel time around the typical value, arising from fluctuations in travel demand, weather conditions, road works etc. (FWHA, 2008). Day-to-day travel time variability forces travelers and transporters to balance the risk of arriving late, including missed appointments, production chain disruption costs etc., against the cost of precautions, such as adding buffers of time and resources. Many modern technologies for traffic and transport management, such as congestion tolls and intelligent traffic information systems, have among their main goals to reduce the variability of travel times.

Considerable attention has been given to assessing the value of travel time variability, VTTV, (or conversely, reliability) for personal travel. One methodological approach, known as the "mean-variance" approach, is to include some measure of the variability, such as the standard deviation or an interpercentile range, directly in individuals' utility functions (e.g., Noland and Polak, 2002; Small et al., 2005; Börjesson et al., 2011). The method does not provide a strong theoretical foundation for the cost (Fosgerau and Karlström, 2010). Another approach, therefore, is to derive the VTTV from a structural model of individuals' travel preferences. The prevailing method is to use a scheduling model for trip departure times, in which costs may arise from early/late departures and/or early/late arrivals. The value of variability is then extracted assuming that the traveler chooses the departure time to minimize the expected cost under the stochastic travel time.

Until now, the VTTV has been assessed considering the scheduling of a single trip in isolation. The most widely used model is the piecewise constant marginal cost model used by Vickrey (1969) and Small (1982). In this model, the travel cost rises proportionally to the earliness of departure and to the lateness or earliness of arrival in relation to a preferred arrival time (sometimes with an additional discrete lateness penalty). From this model, the VTTV can be studied theoretically (e.g., Noland and Small, 1995; Bates et al., 2001) and empirically (e.g., Bates et al., 2001; Asensio and Matas, 2008; Börjesson and Eliasson, 2011). Fosgerau and Karlström (2010) show that the VTTV is independent of the mean and the spread of the travel time distribution but dependent on the shape of its right tail.

A more general scheduling model gaining interest is that of Vickrey (1973), in which the marginal costs of departing earlier or arriving later are general functions of time. Vickrey (1973) showed that the Vickrey (1969) is a special case, while Tseng and Verhoef (2008) estimate a flexible, piecewise constant version of the model for the morning commute. Fosgerau and Engelson (2011) consider another special case with linear marginal utility rates and show that the VTTV, with the travel time variability measured by the standard deviation, is proportional to the standard deviation, which is an attractive feature for network routing and traffic assignment models. Empirical estimation results for the linear model are reported by Börjesson et al. (2011), who find that the model fits the data better than the Vickrey (1969) model.

Single-trip scheduling models provide many essential insights about the factors behind the cost of travel time variability as well as feasible frameworks for empirical estimation. However, the models only consider a single trip in isolation, assuming that lateness and earliness costs are independent of previous and subsequent trips and activities. Single-trip models do not capture the fact that the trip is typically a link in a daily chain of activities and trips where the scheduling of many activities is flexible, so that the utility derived from them depends on both the time of day and the time spent at the activity. This suggests that earliness and lateness costs on a given trip may be determined not only by the departure and arrival times on the trip itself but also by the arrival time to the preceding activity and the departure time on the subsequent trip. It also implies that the subsequent schedule can be adjusted in response to the arrival time of the trip in order to minimize the costs. The effects of these interdependencies on the value of mean travel time, VMTT, and VTTV on different trips have so far not been investigated.

Furthermore, single-trip scheduling models cannot capture the impact on the VMTT and VTTV when travel times are correlated (or, more generally, dependent) across trips. Poor weather conditions, unusally high travel demand or an incident on a particular day, for example, can cause travel times to be unusually long on several or all links in the trip chain. This persistent variability is often particularly strong in scheduled transport systems such as railway and aviation systems (through, e.g., "knock-on" delay). Persistent variability implies that the VMTT and VTTV will vary not only between trips but also with the realizations of travel times on preceding trips, that is, across days. This fact has not received much notice previously but has important empirical implications for the collection, calculation and interpretation of travel time values.

1.2 Aim and scope of the paper

As a background to the subsequent analysis, the paper first provide formulas for the VMTT and the VTTV for the general single-trip scheduling model of Vickrey (1973). The mean and standard deviation of the travel time are allowed to depend on the departure time. The general formulas provide insights into the properties of the VMTT and VTTV that specific functional forms cannot. Furthermore, the formulas for any special case can be directly obtained from the general formulas, whether closed form expressions for the optimal departure time exist or numerical calculations are necessary. We illustrate this fact by deriving the previously known VMTT and VTTV for the special cases with piecewise constant and linear marginal utility rates (Fosgerau and Karlström, 2010; Fosgerau and Engelson, 2011).

The main contribution of this paper is to generalize the single-trip modeling framework for the analysis of the VMTT and VTTV into a multi-trip model, taking into account flexible activity scheduling and dependent travel times. This represents an extension of the simple but analytically tractable single-trip models into the field of activity-based travel demand modeling (Axhausen and Gärling, 1992; Bowman and Ben-Akiva, 2001; Arentze and Timmermans, 2009). Furthermore, the stochasticity of travel times brings the analysis into the framework of multi-stage stochastic programming (e.g., Ruszczyński and Shapiro, 2003). The model is a generalization of the two-trip model of Jenelius et al. (2011) (based in turn on the model of Ettema and Timmermans (2003)), which was used to assess the value of travel time and the costs of delays in a setting of deterministic travel times.

We derive general formulas for the VMTT and VTTV on any trip and show that they depend on the travel times on preceding trips in two distinct ways. First, the scheduling flexibility of activities means that the optimal timing of a trip depends on the arrival time to the preceding activity, which, taken across the distribution of all preceding travel times, is a stochastic variable. Second, the preceding travel time realizations themselves provide information about the subsequent travel times, which also determines the optimal timing of a trip. We define the unconditional VMTT and VTTV as the expected values across all realizations of preceding travel times and show that the model simplifies to the single-trip model when activity scheduling is fixed and travel times are statistically independent. We also outline sufficient conditions on activity utilities and travel time distributions for extending the single-trip analysis approach to the multi-trip setting.

Finally we analyze two special cases of the multi-trip model with piecewise constant and linear marginal utility rates, respectively. For these special cases we obtain instructive closed-form expressions for the VMTT and VTTV on each trip, which are compared with the results for the corresponding single-trip models.

The paper is organized as follows. Section 2 reviews the single-trip model and derives optimality conditions and formulas for the VMTT and VTTV. In Section 3 we present the multi-trip model with flexible scheduling, analyze the necessary conditions for extending the single-trip model to trip chains, and derive the corresponding optimality conditions and formulas for the VMTT and VTTV on any trip. Sections 4 and 5 analyze the two-trips, piecewise constant and linear special cases, respectively, and Section 6 concludes.

2 A single-trip model

In this section we consider a daily schedule that consists of two activities and an intermediate trip. The utility derived from an activity is assumed to be independent of other activities, so that the total utility is additively separable in time spent at each activity. We fix two times $t = t_s$ and $t = t_e$ that mark the start and end of the period of interest, respectively. The schedule can then be uniquely represented by the departure time from activity 1, denoted s. The utility gained from spending another unit of time on activity $i \in \{1, 2\}$ at time t is expressed in the form of a deterministic marginal utility function $u_i(t)$; $u_1(t)$ is continuous while $u_2(t)$ is allowed to be piecewise continuous. We assume that there is a point in time, normalized to $t = 0$, where the marginal utilities intersect, so that the traveler prefers to be at activity 1 before $t = 0$ and at activity 2 after $t = 0$. The marginal utility derived from traveling, denoted ν, is assumed to be constant.[1]

Absolute levels of utility are unobservable, and operational models must be based on differences in utility relative to some baseline level. The models remain equivalent regardless of this choice, however. Throughout this paper we use the marginal utility of travel, ν, as the reference level. Assuming that utility is

[1]The assumption that ν is constant is not strictly necessary here but conforms to the multi-trip model in Section 3, where it is useful.

money metric, we define $c_i(t) \equiv u_i(t) - \nu$, $i \in \{1, 2\}$, as the marginal cost of travelling rather than taking part in activity i at time t.

The time required to travel from activity 1 to 2 is stochastic and depends in general on the departure time s, $T(s) > 0$. Specifically, we assume that $T(s) = \tau(s) + \sigma(s)X$, where X is the standardized travel time, independent of s, with mean zero, standard deviation 1 and cumulative distribution function Φ.[2] The support of Φ is an interval $[\underline{x}, \overline{x}]$ so that the inverse Φ^{-1} exists. The mean $E[T(s)] = \tau(s) > 0$ and the standard deviation $\sigma(s) > 0$ are continuously differentiable functions of s on $[t_s, t_e]$.

Following Tseng and Verhoef (2008) it is convenient to consider the cost relative to the utility derived from an optimal schedule when travel is instantaneous, i.e., when $T \equiv 0$. Since travel time is stochastic, so is the cost, and travelers are assumed to minimize *expected* cost,

$$\min_s \; E[C(s)] \equiv \int_s^0 c_1(t)dt + E\left[\int_0^{s+T(s)} c_2(t)dt\right]. \tag{1}$$

A feasible departure time is one such that $s \geq t_s$ and $s + T(s) \leq t_e$ almost surely.

2.1 Single-trip model: optimality conditions

A marginal change in the departure time affects the time spent in activity 1, the time spent traveling and the time spent in activity 2, and the necessary optimality condition for an interior optimal departure time requires the net effect on the expected cost be zero. The expectation operator smooths the piecewise continuous marginal cost $c_2(t)$ and makes $E[C(s)]$ continuous. Hence, the first-order condition for departure time s^* is

$$0 = \frac{\partial E[C(s)]}{\partial s} = \frac{\partial}{\partial s}\left(\int_s^0 c_1(t)dt + E\left[\int_0^{s+T(s)} c_2(t)dt\right]\right).$$

Since the marginal utilities and the travel time distribution are well-behaved, we can move the differentiation inside the expectation, to obtain

$$c_1(s^*) = E[(1 + T'(s^*)) \cdot c_2(s^* + T(s^*))], \tag{2}$$

where $T'(s) = \tau'(s) + \sigma'(s)X$. As illustrated in Figure 1, the optimality condition can be interpreted as finding the intersection of the functions $c_1(t)$ and $E[(1 + T'(t)) \cdot c_2(t + T(t))]$. An important special case is when travel time is independent of departure time, that is, $T = \tau + \sigma X$ for all s. In this case the optimality condition simplifies to

$$c_1(s^*) = E[c_2(s^* + T)]. \tag{3}$$

The existence of an interior optimum depends on the shapes of the marginal utility functions and the travel time function. It is guaranteed, for example, if $c_1(t)$ is continuous and decreasing, $c_2(t)$ is increasing and $\tau(s)$ and $\sigma(s)$ vary sufficiently slowly with the departure time s, in which case problem (1) is continuous and convex.

[2] The invariance of the standardized travel time distribution over time is found empirically to be a good approximation by Fosgerau and Fukuda (2010).

[**Figure 1 about here.**]

Figure 1: Illustration of the single-trip model and the optimal departure time. The curved, dashed line at time a indicates that the arrival time is stochastic.

2.2 Single-trip model: value of mean travel time and variability

Let us define the marginal value of mean travel time (VMTT) as the marginal expected cost of a uniform increase in the mean travel time given that the departure time is chosen optimally. Without loss of generality, we write the mean travel time as $\tau(s) = \tau_0 + \tau_x(s)$, where $\tau_0 > 0$, and consider changes in the fixed component τ_0. Due to the limited time available in a day, a change of τ_0 affects the amounts of time spent in the two activities. Since the departure time is chosen optimally, however, the net marginal effect on expected cost of the changed departure time is zero. According to the envelope theorem (e.g., Mas-Colell et al., 1995), it is sufficient to consider the partial effect of the changed travel time on activity 2. The value of mean travel time is given by

$$\begin{aligned}\text{VMTT} \equiv \frac{dE[C(s^*)]}{d\tau_0} &= \frac{\partial}{\partial \tau_0} E\left[\int_0^{s^*+\tau_0+\tau_x(s^*)+\sigma(s^*)X} c_2(t)dt\right] \\ &= E[c_2(s^* + T(s^*))].\end{aligned} \tag{4}$$

Some immediate observations can be made. First, the VMTT is independent of the travel time distribution if $c_2(t)$ is constant. Further, if travel time is independent of departure time, optimality condition (3) implies VMTT $= c_1(s^*)$. In this case the VMTT is independent of the travel time distribution if $c_1(t)$ is constant.

Similarly to the VMTT, we can define the marginal value of travel time variability (VTTV) as the marginal expected cost of a uniform increase in the travel time standard deviation given that the departure time is chosen optimally. We write the standard deviation of travel time as $\sigma(s) = \sigma_0 + \sigma_x(s)$, where $\sigma_0 > 0$, and consider changes in the fixed component σ_0. The value of travel time variability is

$$\begin{aligned}\text{VTTV} \equiv \frac{dE[C(s^*)]}{d\sigma_0} &= \frac{\partial}{\partial \sigma_0} E\left[\int_0^{s^*+\tau(s^*)+[\sigma_0+\sigma_x(s^*)]X} c_2(t)dt\right] \\ &= E[X \cdot c_2(s^* + T(s^*))],\end{aligned} \tag{5}$$

where again the envelope theorem was applied. Note that the absolute level of $c_2(t)$ does not affect the VTTV since $E[X] = 0$. Another measure of travel time variability that is sometimes used is the variance. It follows as a corollary to (5) that the value of travel time variance is $dE[C(s^*)]/d\sigma_0^2 = 1/(2\sigma_0)\text{VTTV} = 1/(2\sigma_0)E[X \cdot c_2(s^* + T(s^*))]$.

Formulas (4) and (5) provide a general and direct way to calculate the VMTT and VTTV for the single-trip scheduling model, whether closed form expressions exist for the optimal departure time or numerical calculations are necessary. Using these formulas avoids an explicit derivation of the expected cost function and its value at the optimal departure time. In the following we apply them to two important special cases for which analytical results are known; multi-trip extensions of these models will be considered in Section 3.

2.3 Special case: the step model

This model was proposed by Vickrey (1969) and recently analyzed by Fosgerau and Karlström (2010). Here we show that it represents a special case of the general model above. Using the well-known (α, β, γ) parameterization of Small (1982) for the cost of travel time, schedule delay early and schedule delay late, respectively, marginal costs relative to travel are given by

$$c_1(t) = \alpha,$$
$$c_2(t) = \alpha - \beta + (\beta + \gamma)H(t), \quad t \in [t_s, t_e],$$

where $-\beta < 0 < \gamma$ and $H(t)$ is the Heaviside step function defined as $H(t) = 0$ for $t \leq 0$ and $H(t) = 1$ for $t > 0$.

We here consider the case where travel time is independent of the departure time. The general first-order condition (3) becomes $\alpha = E[\alpha - \beta + (\beta + \gamma)H(s^* + T)]$. Noting that $E[H(s^* + T)] = \Pr(s^* + T \geq 0) = 1 - \Phi((-s^* - \tau)/\sigma)$, we can solve for the optimal departure time $s^* = -\tau - \sigma\Phi^{-1}(\gamma/(\beta + \gamma))$.

The general results in Section 2.2 and the fact that $c_1(t)$ is constant immediately gives VMTT $= \alpha$, which is independent of the travel time distribution. According to formula (5) for the VTTV we then have

$$\begin{aligned} \text{VTTV} &= E[X \cdot [\alpha - \beta + (\beta + \gamma)H(s^* + T)]] \\ &= (\beta + \gamma) \int_{\frac{\gamma}{\beta+\gamma}}^{1} \Phi^{-1}(p)dp, \end{aligned}$$

independent of τ and σ, as previously found by Fosgerau and Karlström (2010).

2.4 Special case: the slope model

This model was recently analyzed by Fosgerau and Engelson (2011). Marginal costs are given by

$$c_1(t) = \beta_0 + \beta_1 t,$$
$$c_2(t) = \gamma_0 + \gamma_1 t, \quad t \in [t_s, t_e],$$

where $\beta_1 < \gamma_1$. The normalization of time so that $c_1(0) = c_2(0)$ implies $\beta_0 = \gamma_0$.

Assuming that travel time is independent of departure time, the general first-order condition (3) requires that $\gamma_0 + \beta_1 s^* = E[\gamma_0 + \gamma_1(s^* + T)]$. We can solve for the optimal departure time $s^* = -\gamma_1/(\gamma_1 - \beta_1) \cdot \tau$ and obtain VMTT $= \gamma_0 + \beta_1 s^* = \gamma_0 - \beta_1\gamma_1/(\gamma_1 - \beta_1) \cdot \tau$. The VMTT thus increases proportionally to the mean travel time, unless $\gamma_1 = 0$ or $\beta_1 = 0$ (constant marginal cost at the destination or the origin), in which cases it is constant, or $\gamma_1 < 0$, in which case it decreases. Using formula (5) we further obtain

$$\text{VTTV} = E[X \cdot (\gamma_0 + \gamma_1(s^* + T))] = \gamma_1\sigma,$$

proportional to σ while independent of τ and any other characteristics of the travel time distribution. In other words, the value of travel time variance, $1/(2\sigma)$VTTV $= \gamma_1/2$ is constant, as found by Fosgerau and Engelson (2011).

3 A multi-trip model with flexible scheduling and dependent travel times

In this section we extend the single-trip model in Section 2 and consider a schedule that consists of $n+1$ activities and a chain of $n \geq 1$ intermediate trips. We generalize the single-trip formulas for the VMTT and the VTTV, taking into account the dependencies between activities and trips. We also state sufficient conditions under which the single-trip analysis can be generalized to the multi-trip setting in this way. The model is an extension of the three activities, two trips model of Jenelius et al. (2011) (which in turn is based on a formulation of Ettema and Timmermans (2003)), who showed that partially flexible scheduling implies that the value of deterministic travel time (VTT) varies between different trips. The introduction of stochastic travel times brings the model into the framework of multi-stage stochastic programming (e.g., Ruszczyński and Shapiro, 2003).

3.1 Framework

3.1.1 Activities

We assume that the number and order of activities and trips are already determined and fixed; that is, we do not consider reordering, replacing, adding or cancelling of activities. The activities are then numbered $i = 1, \ldots, n+1$ in chronological order. To fix ideas, we let the time period covered by the schedule represent a single day. As before, the utility derived from taking part in an activity is assumed to be independent of other activities. Further, the scheduling of a day is independent of preceding and subsequent days. This represents the boundary constraints of the problem and is not a critical assumption, but it facilitates the analysis (for discussions on the representation of day-to-day dependencies in activity-based models, see, e.g., Karlström (2005); Arentze et al. (2010)).

For the first and last activities the utility gained from spending another unit of time at time t is represented by a deterministic marginal utility function $u_i(t)$. For intermediate activities $i \in \{2, \ldots, n\}$ we assume that the marginal utility at time t is a function of a linear combination of the time of day t and the duration $t - a_{i-1}$ since the arrival at a_{i-1}, i.e., $u_i(t - \xi_i a_{i-1})$, where $\xi_i \in [0, 1]$ is a parameter expressing the scheduling flexibility of the activity (Ettema and Timmermans, 2003; Jenelius et al., 2011). All marginal utility functions are piecewise continuously differentiable. It will be notationally convenient in the following to define $\xi_1 \equiv 0$, $a_0 \equiv t_s$ and $\xi_{n+1} \equiv 0$, reflecting that the scheduling of the first and the last activity is fixed.

Note that $\xi_i = 0$ means that marginal utility depends only on time of day, while $\xi_i = 1$ means that marginal utility depends only on time since arrival, i.e., activity duration. Flexibility here thus refers to the degree to which the utility of taking part in an activity is independent of the time of day and is not associated with any particular assumption about the shape of the marginal utility function.

To ensure that the traveler wants to spend some time at each activity, the marginal utility rates of consecutive activities must intersect. Given the arrival time to the origin activity a_{i-1}, there is some intersection time point $t_i^* =$

$t_i^*(a_{i-1})$ such that the traveler prefers to be at the origin activity at $t < t_i^*$, and at the destination activity at $t > t_i^*$. We return to what this entails for the marginal utility functions in Section 3.4.

Importantly, the marginal utility of travel, ν, is assumed to be constant across all trips. With a marginal utility of travel varying with time or with the duration of individual trips a traveler would trade off travel time on different trips, which would complicate the analysis.

As in the single-trip model we use money-metric utility with ν as the reference level and define $c_i(t) \equiv u_i(t) - \nu$, $i \in \{1, \dots, n+1\}$. We find it more convenient here to phrase the problem in terms of the utility relative to traveling the entire day rather than the cost relative to the situation with instantaneous travel. Note that an increase in utility is equivalent to a corresponding decrease in cost.

3.1.2 Trips

Trip i is between activity i and $i+1$. The travel time on each trip is stochastic and depends on the departure time s_i. Anologously to the single-trip model, we assume that travel time can be decomposed as $T_i(s_i, X_i) = \tau_i(s_i) + \sigma_i(s_i)X_i$, where X_i is the (marginal) standardized travel time with mean zero, standard deviation 1 and cumulative distribution function Φ_i for all s_i. The standardized travel times on different trips are in general not independent.

The support of Φ_i is an interval $[\underline{x}_i, \overline{x}_i]$ so that the inverse Φ_i^{-1} exists. The mean $E[T_i(s_i, X_i)] = \tau_i(s_i) > 0$ and the standard deviation $\sigma_i(s_i) > 0$ of each trip travel time are twice continuously differentiable functions of s_i on $[t_s, t_e]$. Since travel time must be non-negative, we require that $\tau_i(s_i) + \sigma_i(s_i)\underline{x}_i \geq 0$ for all s_i. To ensure that all trips can be made within the day for at least some choice of departure times, $\overline{x}_i$ must be finite. Further constraints on the travel time distributions to facilitate the analysis are discussed in Section 3.4.

3.2 The multi-trip scheduling problem

We refer to a vector of departure times $\mathbf{s} = (s_1, \dots, s_n)$ as a *schedule* for short. The arrival time on trip i given departure time s_i and travel time x_i is denoted $a_i(s_i, x_i) \equiv s_i + T_i(s_i, x_i)$. A departure time can be chosen conditional on travel times and departure times on preceding trips but not on subsequent trips (the so-called non-anticipatory constraint of stochastic programming). If the entire problem is seen from the perspective of the start of the day, each decision variable except the first departure time is not a specific time of day but a *mapping* from the probability space of preceding travel times to the space of feasible departure times. We can approach the problem using dynamic programming, however, so that each decision variable is chosen when all relevant stochastic factors have been observed.

Thus, the traveler chooses the schedule sequentially; specifically, the departure time of a trip $i = 2, \dots, n$ is chosen once the traveler has arrived to the preceding activity, having observed the arrival time a_{i-1} and the realized standardized travel times $\mathbf{x}_{1:i-1} \equiv (x_1, \dots, x_{i-1})$, i.e., an outcome of $\mathbf{X}_{1:i-1} = (X_1, \dots, X_{i-1})$ on the preceding trips. The process can be expressed as (c.f. Ruszczyński and Shapiro, 2003):

$$\begin{aligned} &\text{decision } (s_1) \to \text{observation } (x_1) \to \text{decision } (s_2) \to \cdots \\ &\qquad\qquad \to \text{observation } (x_{n-1}) \to \text{decision } (s_n) \ . \end{aligned}$$

For the notation it is convenient to also introduce the quantity x_0 that is observed prior to the departure of the first trip. This may represent prior information about the travel times during the day based on experiences from preceding days or other available information such as weekday, time of year etc.

The expected utility associated with a schedule can be written as a sequence of nested expectations conditional on the realized travel times preceding each trip,

$$\begin{aligned} E[U(\mathbf{s}) \,|\, x_0] = \int_{t_s}^{s_1} c_1(t)dt + E\Bigg[\int_{(1-\xi_2)a_1(s_1,X_1)}^{s_2-\xi_2 a_1(s_1,X_1)} c_2(t)dt + \cdots \\ + E\Bigg[\int_{(1-\xi_n)a_{n-1}(s_{n-1},X_{n-1})}^{s_n-\xi_n a_{n-1}(s_{n-1},X_{n-1})} c_n(t)dt \\ + E\Bigg[\int_{a_n(s_n,X_n)}^{t_e} c_{n+1}(t)dt \,\Bigg|\, \mathbf{X}_{0:n-1}\Bigg] \,\Bigg|\, \mathbf{X}_{0:n-2}\Bigg] \cdots \,\Bigg|\, x_0\Bigg]. \end{aligned}$$

A feasible schedule is one such that $s_1 \geq t_s$, $s_i \geq a_{i-1}(s_{i-1}, x_{i-1})$ for $i \in \{2, \ldots, n\}$, and $a_n(s_n, X_n) \leq t_e$ almost surely.

3.3 Multi-trip model: optimality conditions

Much like for the single-trip model, we focus on interior solutions, that is, we assume that the traveler spends a positive amount of time in each activity for any outcome of the travel times. We consider boundary solutions, corresponding to departing on the next trip immediately after arriving, to be situations of limited practical interest. Other adjustments, not covered here, seem more reasonable in such severe situations, for example cancelling some of the subsequent activities. In Section 3.4 we present sufficient conditions on the marginal utilities and the travel time distributions to guarantee interior solutions.

3.3.1 The last trip

Using a dynamic programming approach, we first consider the last trip, indexed n. An optimal departure time $s_n^* = s_n^*(a_{n-1}, \mathbf{x}_{0:n-1})$ is a solution to the problem

$$\max_{s_n} \tilde{U}_n(s_n, a_{n-1}, \mathbf{x}_{0:n-1}) \equiv \int_{(1-\xi_n)a_{n-1}}^{s_n-\xi_n a_{n-1}} c_n(t)dt + E\left[\int_{a_n(s_n,X_n)}^{t_e} c_{n+1}(t)dt \,\middle|\, \mathbf{x}_{0:n-1}\right] \tag{6}$$

Thus, if the scheduling of the preceding activity is at least partially flexible ($\xi_n > 0$), the optimal departure time s_n^* depends on the arrival time a_{n-1} to the preceding activity. Further, it depends in general on the realized standardized travel times on all preceding trips, $\mathbf{x}_{0:n-1}$, since these observations provide a signal for X_n through the joint travel time distribution.

A marginal change in the departure time s_n affects the time spent in activity n, the time spent traveling on trip n and the time spent in activity $n+1$, and the first-order condition for an interior optimum requires that the net change in expected utility be zero. More precisely, the first-order condition for the departure time is

$$
\begin{aligned}
0 &= \frac{\partial}{\partial s_n} \tilde{U}(s_n, a_{n-1}, \mathbf{x}_{0:n-1}) \\
&= \frac{\partial}{\partial s_n} \left(\int_{(1-\xi_n)a_{n-1}}^{s_n - \xi_n a_{n-1}} c_n(t)dt + E\left[\int_{a_n(s_n, X_n)}^{t_e} c_{n+1}(t)dt \,\middle|\, \mathbf{x}_{0:n-1} \right] \right),
\end{aligned}
$$

that is, with $a'_n(s_n, X_n) = 1 + \tau'_n(s_n) + \sigma'_n(s_n)X_n$,

$$
c_n(s^*_n - \xi_n a_{n-1}) = E[a'_n(s^*_n, X_n) \cdot c_{n+1}(a_n(s^*_n, X_n)) \,|\, \mathbf{x}_{0:n-1}].
$$

Problem (6) is thus essentially equivalent to the single-trip scheduling problem (1).

3.3.2 The second to last trip

Consider now the preceding trip $n-1$ and assume that the departure time of trip n, s^*_n, is chosen optimally given the preceding arrival time and the realized travel times on all preceding trips. s^*_n is thus a function of s_{n-1}: at time $n-1$, it is known only as a stochastic variable $s^*_n = s^*_n(a_{n-1}(s_{n-1}, X_{n-1}), \mathbf{x}_{0:n-2}, X_{n-1})$, while at time n, it is known as the deterministic time $s^*_n = s^*_n(a_{n-1}(s_{n-1}, x_{n-1}), \mathbf{x}_{0:n-1})$. An optimal departure time $s^*_{n-1} = s^*_{n-1}(a_{n-2}, \mathbf{x}_{0:n-2})$ is a solution to the problem

$$
\begin{aligned}
\max_{s_{n-1}} \tilde{U}_{n-1}&(s_{n-1}, a_{n-2}, \mathbf{x}_{0:n-2}) \\
\equiv& \int_{(1-\xi_{n-1})a_{n-2}}^{s_{n-1} - \xi_{n-1} a_{n-2}} c_{n-1}(t)dt + E\Bigg[\int_{(1-\xi_n)a_{n-1}(s_{n-1}, X_{n-1})}^{s^*_n - \xi_n a_{n-1}(s_{n-1}, X_{n-1})} c_n(t)dt \\
&+ E\left[\int_{a_n(s^*_n, X_n)}^{t_e} c_{n+1}(t)dt \,\middle|\, \mathbf{x}_{0:n-2}, X_{n-1} \right] \Bigg|\, \mathbf{x}_{0:n-2} \Bigg] \\
=& \int_{(1-\xi_{n-1})a_{n-2}}^{s_{n-1} - \xi_{n-1} a_{n-2}} c_{n-1}(t)dt \\
&+ E\big[\tilde{U}_n(s^*_n, a_{n-1}(s_{n-1}, X_{n-1}), \mathbf{x}_{0:n-2}, X_{n-1}) \,\big|\, \mathbf{x}_{0:n-2}\big]
\end{aligned}
$$

Note how the optimal solution for the last trip enters the scheduling problem of the preceding trip.

A marginal change in the departure time s_{n-1} affects the times spent in activities $n-1$ and n directly, and the times spent in activities n and $n+1$ through the induced change in subsequent departure time s^*_n. The first-order condition for an interior condition requires that the net change in expected utility be zero,

$$
\begin{aligned}
0 &= \frac{d}{ds_{n-1}} \tilde{U}_{n-1}(s_{n-1}, a_{n-2}, \mathbf{x}_{0:n-2}) \\
&= c_{n-1}(s_{n-1} - \xi_{n-1} a_{n-2}) \\
&\quad + \frac{d}{ds_{n-1}} E\big[\tilde{U}_n(s^*_n, a_{n-1}(s_{n-1}, X_{n-1}), \mathbf{x}_{0:n-2}, X_{n-1}) \,\big|\, \mathbf{x}_{0:n-2}\big].
\end{aligned}
$$

Under our regularity conditions on the marginal utility and travel time functions, the order of the expectation and differentiation operators can be switched. Now, since s_n^* by assumption is an interior maximum of $\tilde{U}_n(s_n, a_{n-1}(s_{n-1}, x_{n-1}), \mathbf{x}_{0:n-1})$, the net change in expected utility due to this induced departure time change, $d\tilde{U}_n/ds_n \cdot ds_n^*/ds_{n-1}$, will be zero for *any* realization x_{n-1}. Therefore, the *expected* net change across all possible realizations X_{n-1} is also zero, and by the envelope theorem it is sufficient to consider only the direct effect of a change in s_{n-1} on expected utility, i.e., on the times spent in activities $n-1$ and n. The necessary optimality condition is thus

$$
\begin{aligned}
0 &= \frac{\partial}{\partial s_{n-1}} \tilde{U}_{n-1}(s_{n-1}, a_{n-2}, \mathbf{x}_{0:n-2}) \\
&= c_{n-1}(s_{n-1} - \xi_{n-1} a_{n-2}) + \frac{\partial}{\partial s_{n-1}} E\left[\int_{(1-\xi_n)a_{n-1}(s_{n-1}, X_{n-1})}^{s_n^* - \xi_n a_{n-1}(s_{n-1}, X_{n-1})} c_n(t)dt \,\middle|\, \mathbf{x}_{0:n-2} \right].
\end{aligned}
$$

It is convenient here to introduce the *backward optimal* marginal cost function $\tilde{c}_n(a_{n-1}, \mathbf{x}_{0:n-1})$ (a generalization of the backward optimal utility function of Jenelius et al. (2011)), defined as the marginal cost of an increase in the arrival time to activity n given that s_n^* is an interior optimum. By the envelope theorem it is sufficient to consider the partial effect of increasing a_{n-1}, i.e.,

$$
\begin{aligned}
\tilde{c}_n(a_{n-1}, \mathbf{x}_{0:n-1}) &\equiv -\frac{d}{da_{n-1}} \tilde{U}_n(s_n^*, a_{n-1}, \mathbf{x}_{0:n-1}) \\
&= -\frac{\partial}{\partial a_{n-1}} \int_{(1-\xi_n)a_{n-1}}^{s_n^* - \xi_n a_{n-1}} c_n(t)dt \\
&= (1-\xi_n) c_n((1-\xi_n)a_{n-1}) + \xi_n c_n(s_n^*(a_{n-1}, \mathbf{x}_{0:n-1}) - \xi_n a_{n-1}).
\end{aligned}
$$

The optimality condition can now be written as

$$
\begin{aligned}
&c_{n-1}(s_{n-1}^* - \xi_{n-1} a_{n-2}) = \\
&E\big[a'_{n-1}(s_{n-1}^*, X_{n-1}) \cdot \tilde{c}_n(a_{n-1}(s_{n-1}^*, X_{n-1}), \mathbf{x}_{0:n-2}, X_{n-1}) \,\big|\, \mathbf{x}_{0:n-2}\big].
\end{aligned}
$$

Note the similarity with the optimality condition for the last trip above.

3.3.3 A general trip

The same line of reasoning as for trip $n-1$ can be extended to any trip $i \in \{1, n-1\}$. Assume that the departure times on all trips subsequent to i are interior optima given the arrival time to the immediately preceding activity and the realized travel times on all preceding trips. An optimal departure time $s_i^* = s_i^*(a_{i-1}, \mathbf{x}_{0:i-1})$ is then a solution to the problem

$$
\begin{aligned}
\max_{s_i} \; \tilde{U}_i(s_i, a_{i-1}, \mathbf{x}_{0:i-1}) \equiv & \int_{(1-\xi_i)a_{i-1}}^{s_i - \xi_i a_{i-1}} c_i(t)dt \\
& + E\big[\tilde{U}_{i+1}(s_{i+1}^*, a_i(s_i, X_i), \mathbf{x}_{0:i-1}, X_i) \,\big|\, \mathbf{x}_{0:i-1}\big].
\end{aligned}
$$

By backward induction, making repeated use of the same arguments as for trip $n-1$ above and the law of iterated expectations, it follows that the necessary optimality condition for an interior optimum requires the partial effect on expected

[**Figure 2 about here.**]

Figure 2: Illustration of a trip i in the multiple-trip model and the optimal departure time s_i^*. The double, dashed lines for $\tilde{c}_{i+1}(t, (\mathbf{x}_{0:i-1}, X_i))$ indicates that the backward optimal marginal cost function is stochastic. The curved, dashed line at time a_i indicates that the arrival time is stochastic.

utility to be zero, i.e., $\partial\tilde{U}_i(s_i, a_{i-1}, \mathbf{x}_{0:i-1})/\partial s_i = 0$. Generalizing the definition of the backward optimal marginal cost function to any activity $i \in \{2, \ldots, n+1\}$ as

$$\tilde{c}_i(a_{i-1}, \mathbf{x}_{0:i-1}) = (1-\xi_i)c_i((1-\xi_i)a_{i-1}) + \xi_i c_i(s_n^*(a_{i-1}, \mathbf{x}_{0:i-1}) - \xi_i a_{i-1}), \quad (7)$$

the necessary optimality conditions are

$$c_i(s_i^* - \xi_i a_{i-1}) = E\big[a_i'(s_i^*, X_i) \cdot \tilde{c}_{i+1}(a_i(s_i^*, X_i), \mathbf{x}_{0:i-1}, X_i) \,\big|\, \mathbf{x}_{0:i-1}\big], \quad i \in \{1, \ldots, n\}. \quad (8)$$

Note that we have $\tilde{c}_{n+1}(a_n, \mathbf{x}) = c_{n+1}(a_n)$, consistent with the fixed scheduling of the last activity.

For each trip i optimality condition (8) states that the marginal cost from departing earlier from the origin activity i (left-hand side) must equal the expected marginal cost from arriving later to the destination activity $i+1$ given that the subsequent schedule is chosen optimally, considering that the arrival time is affected by the departure time and is stochastic at the time of departure (right-hand side). This illustrated in Figure 2.

An important special case is when travel time is independent of departure time, that is, $T_i = \tau_i + \sigma_i X_i$ for all s_i. In this case the optimality condition simplifies to

$$c_i(s_i^* - \xi_i a_{i-1}) = E\big[\tilde{c}_{i+1}(a_i(s_i^*, X_i), \mathbf{x}_{0:i-1}, X_i) \,\big|\, \mathbf{x}_{0:i-1}\big], \quad i \in \{1, \ldots, n\}. \quad (9)$$

We may compare Eqs. (8)–(9) with the corresponding Eqs. (2)–(3) for the single-trip model. Given that the conditions for the existence of an interior solution are fulfilled, the optimality conditions for the multi-trip model extends those of the single-trip model in two distinct ways. First, the joint distribution of travel times means that the optimal scheduling of a trip takes the realized travel times on preceding trips into account. Second, the scheduling flexibility of activities means that the optimal timing of a trip depends on the arrival time to the preceding activity. Together, this means that the backward optimal marginal cost function itself is stochastic at the time of departure, which is in contrast to the fixed destination marginal cost function $c_2(t)$ in the single-trip model. If travel times are statistically independent (i.e., $X_i \,|\, \mathbf{x}_{0:i-1} = X_i$ for all trips), and if the scheduling of the activities is completely fixed (i.e., $\xi_i = 0$ for all trips), the optimality condition (8) becomes

$$c_i(s_i^*) = E[a_i'(s_i^*, X_i) \cdot c_{i+1}(a_i(s_i^*, X_i))], \quad i \in \{1, \ldots, n\},$$

which is the same condition as Eq. (2) for the single-trip model. In this case the multi-trip model thus separates into n single-trip model instances.

[Figure 3 about here.]

Figure 3: Illustration of the sufficient conditions for the existence of an interior solution, which must hold simultaneously for all trips.

3.4 Sufficient conditions for interior solutions

The shapes and the interplay of the marginal utility functions and the travel time functions may be arbitrarily complex while still permitting interior solutions to exist jointly for all trips and hence making the multi-trip extension of the single-trip analysis above valid. We will here consider some sufficient (but not necessary) conditions, in addition to the general assumptions in Section 3.1, under which this is guaranteed.

First, we assume that $c_1(t)$ is decreasing, while $c_{n+1}(t)$ is increasing. For each intermediate activity $i \in \{2, \ldots, n\}$, the marginal cost $c_i(t-\xi_i a_{i-1})$ reaches a maximum at some $\hat{t}_i$, i.e., at time of day $t = \hat{t}_i + \xi_i a_{i-1}$, before which it is increasing, representing a warm-up period, and after which it is decreasing, representing a cool-down period. Furthermore, all marginal cost functions are continuously differentiable. Second, we assume that that travel time is independent of departure time, i.e., that $T_i(s_i) = \tau_i + \sigma_i X_i$ for all s_i and all trips i. Later, we argue that the sufficient conditions remain valid if the dependence on departure time is sufficiently weak.

It is shown in the Appendix that under these conditions the backward optimal marginal cost functions $\tilde{c}_{i+1}(t, \mathbf{x}_{0:i})$ are continuously differentiable and increasing in t up to at least $t = \hat{t}_{i+1}/(1 - \xi_i)$, given that the traveler arrives during the warm-up period and departs during the cool-down period of each subsequent activity (not considering at the moment if this is feasible). Hence, if the traveler departs from activity i during the cool-down period and arrives during the warm-up period of activity $i+1$ as defined by the backward optimal marginal cost function $\tilde{c}_{i+1}$, the scheduling problem on trip i, with necessary optimality conditions given in (9) above, is convex and continuous, which guarantees an interior solution much like the single-trip model in Section 2.

Consider setting the standardized travel time on every trip i to the upper bound $\overline{x}_i$, i.e., $T_i = \overline{T}_i = \tau_i + \sigma_i \overline{x}_i$ deterministically. In this case, optimal departure times $\overline{s}_i^*$ are actual points in time rather than policies, and associated arrival times $\overline{a}_i^* = \overline{s}_i^* + \overline{T}_i$ are deterministic. In order for an interior solution to exist for every trip in this worst-case scenario, there must be sufficient time between the start of the cool-down period of activity i and the end of the warm-up period of activity $i+1$, simultaneously for all trips i. For each trip, it is the lower of the peaks of the two marginal cost functions c_i and $\tilde{c}_{i+1}$ that sets the constraint: the upper-bound travel time $\overline{T}_i$ must be less than the time between the peak of the lower marginal cost function and the point in time where the other marginal cost function reaches the same value. This condition is illustrated in Figure 3 for the case when $\tilde{c}_{i+1}$ is lower than c_i.

A key observation is that if an interior solution exists for trip i in the worst-case scenario, then it exists with probability one when travel times are stochastic; hence, the condition above is sufficient. To see this, note that the shorter the travel times on preceding trips, the more the arrival time to activity i is pulled back earlier in time; conversely, the shorter the travel times on subsequent trips, the more the departure time from activity $i+1$ can be pushed forward later in time. In other words, the arrival time to activity i will be earlier than the worst-case $\overline{a}^*_{i-1}$, and the departure time from activity $i+1$ will be later than the worst-case $\overline{s}^*_{i+1}$, with probability one. Furthermore, the travel time on trip i will be shorter or equal to $\overline{T}_i$. Thus, since c_i is decreasing and $\tilde{c}_{i+1}$ is increasing and involves the expectation across all possible realizations of subsequent travel times, there will exist an interior solution with probability one for any actual outcomes on preceding trips.

As discussed in the Appendix, the condition that travel time is independent of departure time can be relaxed while the backward optimal marginal cost function $\tilde{c}_{i+1}(a_i(s_i), \mathbf{x}_{0:i})$ is still increasing and continuously differentiable. Furthermore, a sufficiently weak dependence on departure time means that the effect on the optimal choice of departure time is sufficiently small to allow an interior solution also in this case.

3.5 Multi-trip model: value of mean travel time and variability

Now using forward induction, it can be seen that the arrival time to any activity i and subsequent departure time in an optimal schedule are completely determined by the preceding realized travel times $\mathbf{x}_{0:i-1}$ and can be denoted $a^*_{i-1}(\mathbf{x}_{0:i-1})$ and $s^*_i(\mathbf{x}_{0:i-1})$, respectively. The arrival time $a^*_i(\mathbf{x}_{0:i})$ is related to the preceding departure time as $a^*_i(\mathbf{x}_{0:i}) = s^*_i(\mathbf{x}_{0:i-1}) + T_i(s^*_i(\mathbf{x}_{0:i-1}), x_i)$.

As the intuitive generalization from the single-trip model, we define the value of mean travel time (VMTT) on a trip as the marginal expected cost of a uniform increase in the mean travel time given that all departure times, on both preceding and subsequent trips, are chosen optimally. We write the mean travel time on trip $i \in \{1, \dots, n\}$ as $\tau_i(s_i) = \tau^0_i + \tau^{\times}_i(s_i)$, with $\tau^0_i > 0$, and consider changes in the fixed component τ^0_i. The VMTT on trip i will vary depending on the realized travel times on the preceding trips and the information available before the first trip. We may first define the *conditional* VMTT for a given travel time realization $\mathbf{x}_{0:i-1}$, i.e., on a given day. By the law of iterated expectations and the envelope theorem, we have

$$\begin{aligned} \mathrm{VMTT}_i(\mathbf{x}_{0:i-1}) &\equiv -\frac{dE[U(\mathbf{s}^*) \,|\, \mathbf{x}_{0:i-1}]}{d\tau^0_i} \\ &= -\frac{\partial}{\partial \tau^0_i} E\left[\int_{[1-\xi_{i+1}][s^*_i+\tau^0_i+\tau^{\times}_i(s^*_i)+\sigma_i(s^*_i)X_i]}^{s^*_{i+1}-\xi_{i+1}[s^*_i+\tau^0_i+\tau^{\times}_i(s^*_i)+\sigma_i(s^*_i)X_i]} c_{i+1}(t)dt \,\middle|\, \mathbf{x}_{0:i-1} \right] \\ &= E[\tilde{c}_{i+1}(a^*_i(\mathbf{x}_{0:i-1}, X_i), \mathbf{x}_{0:i-1}, X_i) \,|\, \mathbf{x}_{0:i-1}], \end{aligned}$$
$$i \in \{1, \dots, n\}.$$

Taking the expectation across the whole distribution of $\mathbf{X}_{0:i-1}$, i.e., across all possible days, the *unconditional* VMTT is

$$\begin{aligned} \mathrm{VMTT}_i &\equiv E[\mathrm{VMTT}_i(\mathbf{X}_{0:i-1})] \\ &= E[\tilde{c}_{i+1}(a^*_i(\mathbf{X}_{0:i}), \mathbf{X}_{0:i})], \quad i \in \{1, \dots, n\}. \end{aligned} \tag{10}$$

In the important special case where travel time is independent of departure time, optimality condition (9) implies that

$$\mathrm{VMTT}_i(\mathbf{x}_{0:i-1}) = c_i(s^*_i(\mathbf{x}_{0:i-1}) - \xi_i a^*_{i-1}(\mathbf{x}_{0:i-1})),$$

and across all realizations of preceding travel times we have

$$\mathrm{VMTT}_i = E[c_i(s_i^*(\mathbf{X}_{0:i-1}) - \xi_i a_{i-1}^*(\mathbf{X}_{0:i-1}))]. \tag{11}$$

These alternative formulas express the conditional and the unconditional VMTT in terms of the origin activity rather than the destination activity and may be more useful depending on the model specification.

Similarly to the VMTT, we define the value of travel time variability (VTTV) on a trip as the marginal expected cost of a uniform increase in the travel time standard deviation given an optimal schedule. We write the standard deviation of travel time on trip $i \in \{1, \ldots, n\}$ as $\sigma_i(s_i) = \sigma_i^0 + \sigma_i^{\mathrm{x}}(s_i)$, with $\sigma_i^0 > 0$, and consider changes in the fixed component σ_i^0.

Just as the VMTT, the VTTV on trip i will vary depending on the realized travel times on the preceding trips, and we first consider the conditional VTTV for a given travel time realization $\mathbf{x}_{0:i-1}$. We have

$$\begin{aligned}
\mathrm{VTTV}_i(\mathbf{x}_{0:i-1}) &\equiv -\frac{dE[U(\mathbf{s}^*) \,|\, \mathbf{x}_{0:i-1}]}{d\sigma_i^0} \\
&= -\frac{\partial}{\partial \sigma_i^0} E\left[\int_{[1-\xi_{i+1}][s_i^* + \tau_i(s_i^*) + (\sigma_i^0 + \sigma_i^{\mathrm{x}}(s_i^*))X_i]}^{s_{i+1}^* - \xi_{i+1}[s_i^* + \tau_i(s_i^*) + (\sigma_i^0 + \sigma_i^{\mathrm{x}}(s_i^*))X_i]} c_{i+1}(t)dt \,\middle|\, \mathbf{x}_{0:i-1} \right] \\
&= E[X_i \cdot \tilde{c}_{i+1}(a_i^*(\mathbf{x}_{0:i-1}, X_i), \mathbf{x}_{0:i-1}, X_i) \,|\, \mathbf{x}_{0:i-1}],
\end{aligned}$$
$$i \in \{1, \ldots, n\}.$$

Taken across the whole distribution of $\mathbf{X}_{0:i-1}$, the unconditional VTTV is

$$\begin{aligned}
\mathrm{VTTV}_i &\equiv E[\mathrm{VTTV}_i(\mathbf{X}_{0:i-1})] \\
&= E[X_i \cdot \tilde{c}_{i+1}(a_i^*(\mathbf{X}_{0:i}), \mathbf{X}_{0:i})], \quad i \in \{1, \ldots, n\}.
\end{aligned} \tag{12}$$

Formulas (10) and (12) may be compared with their counterparts (4) and (5) for the single-trip special case. Much like the optimality conditions discussed above, there are two distinct features that influences the unconditional VMTT and VTTV that are not present in the single-trip model. First, the scheduling flexibility of activities means that the optimal timing of a trip depends on the arrival time to the preceding activity, which, taken across the distribution of all preceding travel times, is a stochastic variable. Second, the preceding travel time realizations themselves provide information about the subsequent travel times, which also determines the optimal timing of a trip. It follows that with statistically independent travel times ($X_i \,|\, \mathbf{x}_{0:i-1} = X_i$ for all i) and fixed scheduling ($\xi_i = 0$ for all i) of all activities, we obtain $\mathrm{VMTT}_i = E[c_{i+1}(s_i^* + T_i(s_i^*, X_i))]$ and $\mathrm{VTTV}_i = E[X_i \cdot c_{i+1}(s_i^* + T_i(s_i^*, X_i))]$ equivalent to formulas (4)–(5).

4 Special case: Two trip step model

We now consider a special case of the general model in Section 3, which consists of $n = 2$ trips and 3 activities with piecewise constant marginal cost functions. This specification is an extension of the single-trip model studied in Section 2.3.

The only activity with flexible scheduling is activity 2, which means that we can drop the activity index from the scheduling flexibility parameter and simply denote it as ξ. For activity 2, the marginal cost has a positive jump at

[Figure 4 about here.]

Figure 4: Illustration of the two-trip model with piecewise constant marginal costs.

$t = (1-\xi)t_1^* + \xi a_1$ representing a transition from early to late arrival, and a negative jump at $t = t_2^* - \xi t_1^* + \xi a_1$ capturing early/late departure. For activity 3, the marginal cost has a positive jump at $t = t_3^*$ capturing early/late arrival. Extending the (α, β, γ) notation of Small (1982), marginal costs are given by

$$\begin{aligned}
&c_1(t) = \alpha, \\
&c_2(t) = \alpha - \beta_2 + (\beta_2 + \gamma_2^1)H(t - (1-\xi)t_1^*) + (\gamma_2^2 - \gamma_2^1)H(t - (t_2^* - \xi t_1^*)), \\
&c_3(t) = \alpha - \beta_3 + (\beta_3 + \gamma_3)H(t - t_3^*), \quad t \in [t_s, t_e],
\end{aligned}$$

where $-\beta_2 < \gamma_2^1$, $\gamma_2^2 < \gamma_2^1$ and $-\beta_3 < \gamma_3$.

In the analysis we assume that the traveler always arrives to activity 2 before the second, negative step in marginal cost, i.e., that $(1-\xi)a_1 < t_2^* - \xi t_1^*$ for all feasible arrival times a_1. Also, the traveler always departs from activity 2 after the first, positive step in marginal cost, i.e., $s_2 - \xi a_1 > (1-\xi)t_1^*$ for all feasible departure times s_2 and arrival times a_1. We consider here the case where travel times are independent of departure times, i.e., $T_i = \tau_i + \sigma_i X_i$ for $i = 1, 2$.

4.1 Two trip step model: Optimality conditions

For activity 3, the backward optimal marginal cost function defined in (7) is here $\tilde{c}_3(a_2, (x_1, x_2)) = \alpha - \beta_3 + (\beta_3 + \gamma_3)H(a_2 - t_3^*)$. Similar to the single-trip model, we assume that the marginal cost of early (late) arrival at the destination is always lower (higher) than the marginal cost of departure from the origin, that is, $-\beta_3 < \gamma_2^2 < \gamma_2^1 < \gamma_3$.

There are three possible cases for the optimal departure time on trip 2 depending on parameter values and the preceding arrival time: it falls either (i) before, (ii) after or (iii) precisely at the transition from the high departure cost c_2^2 to the low departure cost c_2^3, which occurs at $t = t_2^* + \xi(a_1 - t_1^*)$. Note that the discontinuity at this point represents a violation of the sufficient conditions in Section 3.4, which means that an interior optimum cannot be guaranteed and that special attention to this point is needed.

For cases (i) and (ii) the optimal departure time s_2^* can be found from the first-order condition (9) applied to trip 2,

$$\alpha + \gamma_2^k = E[\alpha - \beta_3 + (\beta_3 + \gamma_3)H(s_2^* + \tau_2 + \sigma_2 X_2 \,|\, x_1 - t_3^*)], \tag{13}$$

where $k = 1$ in case (i) and $k = 2$ in case (ii). In these cases we can solve for the optimal departure time $s_2^* = s_2^*(a_1, x_1)$ (cf. Section 2.3). By equating these with the transition point $t_2^* + \xi(a_1 - t_1^*)$, we can identify the ranges of the arrival time a_1 under which the different cases apply. In total, we obtain

$$s_2^*(a_1, x_1) = \begin{cases} t_3^* - \tau_2 - \sigma_2 \Phi^{-1}_{X_2 \,|\, x_1}\left(\frac{\gamma_3 - \gamma_2^2}{\beta_3 + \gamma_3}\right) & \text{case (ii): } \xi a_1 \le \underline{a}_1(x_1), \\ t_2^* + \xi(a_1 - t_1^*) & \text{case (iii): } \xi a_1 \in (\underline{a}_1(x_1), \overline{a}_1(x_1)], \\ t_3^* - \tau_2 - \sigma_2 \Phi^{-1}_{X_2 \,|\, x_1}\left(\frac{\gamma_3 - \gamma_2^1}{\beta_3 + \gamma_3}\right) & \text{case (i): } \xi a_1 > \overline{a}_1(x_1), \end{cases} \tag{14}$$

∎

where $\underline{a}_1(x_1) = t_3^* - t_2^* + \xi t_1^* - \tau_2 - \sigma_2 \Phi^{-1}_{X_2\,|\,x_1}\left(\frac{\gamma_3 - \gamma_2^2}{\beta_3 + \gamma_3}\right)$ and $\overline{a}_1(x_1) = t_3^* - t_2^* + \xi t_1^* - \tau_2 - \sigma_2 \Phi^{-1}_{X_2\,|\,x_1}\left(\frac{\gamma_3 - \gamma_2^1}{\beta_3 + \gamma_3}\right)$, and $\Phi_{X_2\,|\,x_1}$ is the cumulative distribution function of X_2 conditional on x_1. The optimal departure time is thus a continuous function of the arrival time with two constant regimes and an intermediate linearly increasing regime.

Consider then trip 1. The scheduling of the destination activity is flexible, and the backward optimal marginal cost function defined in (7) is

$$\begin{aligned}\tilde{c}_2(a_1, x_1) = (1-\xi)[\alpha - \beta_2 + (\beta_2 + \gamma_2^1) H(a_1 - t_1^*)] \\ + \xi[\alpha + \gamma_2^1 + (\gamma_2^2 - \gamma_2^1) H(s_2^*(a_1, x_1) - \xi a_1 - (t_2^* - \xi t_1^*))].\end{aligned} \tag{15}$$

The second term here captures that the optimal departure time on trip 2 may be during the high cost regime γ_2^1 or the low cost regime γ_2^2 depending on the realized travel time on trip 1. The general necessary optimality condition (9) applied to the timing of trip 1 requires that $\alpha = E\left[\tilde{c}_2(s_1^* + \tau_1 + \sigma_1 X_1, X_1)\right]$. In general, it seems not possible to express s_1^* in closed form due to the second term in (15). We may consider two special cases, however, in which (I) $s_2^* - \xi a_1 \leq t_2^* - \xi t_1^*$ (early departure), or (II) $s_2^* - \xi a_1 > t_2^* - \xi t_1^*$ (late departure), for *any* feasible arrival time a_1. In these special cases, the backward optimal marginal cost function simplifies to

$$\tilde{c}_2(a_1) = \alpha - (1-\xi)\beta_2 + \xi \gamma_2^k + (1-\xi)(\beta_2 + \gamma_2^1) H(a_1 - t_1^*),$$

where $k = 1$ in case (I) and $k = 2$ in case (II). As can be seen by comparing with Section 2.3, this is in each case the destination marginal cost function of a single-trip step model with cost parameters

$$\begin{aligned}\beta &= (1-\xi)\beta_2 - \xi\gamma_2^k, \\ \gamma &= (1-\xi)\gamma_2^1 + \xi\gamma_2^k.\end{aligned}$$

Restricting attention to the case that $-\beta < 0 < \gamma$, which is the standard assumption of the single trip model, and $\xi < 1$, we find the optimal departure time in each special case as

$$s_1^* = t_1^* - \tau_1 - \sigma_1 \Phi_{X_1}^{-1}\left(\frac{(1-\xi)\gamma_2^1 + \xi\gamma_2^k}{(1-\xi)(\beta_2 + \gamma_2^1)}\right),$$

where Φ_{X_1} is the marginal cumulative distribution function of X_1. Inserting the associated arrival time $a_1^*(x_1) = s_1^* + \tau_1 + \sigma_1 x_1$ into (14) gives the overall optimal departure time on trip 2 in each special case as

$$s_2^*(x_1) = t_3^* - \tau_2 - \sigma_2 \Phi^{-1}_{X_2\,|\,x_1}\left(\frac{\gamma_3 - \gamma_2^k}{\beta_3 + \gamma_3}\right).$$

4.2 Two trip step model: value of mean travel time and variability

Since the optimal departure time on trip 1 is always an interior optimum, we obtain the value of mean travel time from (11) as

$$\text{VMTT}_1 = \alpha,$$

just as in the single-trip model, independent of scheduling flexibility and other parameters. For trip 2, formula (10) gives

$$\mathrm{VMTT}_2 = E[\alpha - \beta_3 + (\beta_3 + \gamma_3) H(s_2^*(X_1) + \tau_2 + \sigma_2 X_2 - t_3^*)],$$

which in general depends on the travel time on trip 1. Under the optimality conditions (13) in special cases (I) and (II), however, it simplifies to

$$\mathrm{VMTT}_2 = \alpha + \gamma_2^k,$$

independent of trip 1.

Inserting the backward optimal marginal cost function from (15) into formula (12), the value of travel time variability on trip 1 is

$$\begin{aligned}\mathrm{VTTV}_1 = E\big[X_1 \cdot \big(\alpha + (1-\xi) \cdot [-\beta_2 + (\beta_2 + \gamma_2^1) \cdot H(s_1^* + \tau_1 + \sigma_1 X_1 - t_1^*)] \\ + \xi \cdot [\gamma_2^1 + (\gamma_2^2 - \gamma_2^1) \cdot H(s_2^*(X_1) - t_2^* - \xi[s_1^* + \tau_1 + \sigma_1 X_1 - t_1^*])]\big)\big].\end{aligned}$$

In parallel with the single-trip step model, there are closed-form formulas for VTTV_1 in special cases (I) and (II),

$$\mathrm{VTTV}_1 = (1-\xi)(\beta_2 + \gamma_2^1) \int_{\frac{(1-\xi)\gamma_2^1 + \xi\gamma_2^k}{(1-\xi)(\beta_2+\gamma_2^1)}}^1 \Phi_{X_1}^{-1}(p) dp.$$

The flexibility parameter ξ thus enters the VTTV_1 linearly as a multiplier, representing the cost of early/late arrival, but also non-linearly in the lower limit of the integral, representing the choice of departure time. When ξ tends to 0 we recover the expression for the single-trip step model, as expected.

Similarly, the unconditional VTTV on trip 2 is

$$\mathrm{VTTV}_2 = E[X_2 \cdot (\alpha - \beta_3 + (\beta_3 + \gamma_3) H(s_2^*(X_1) + \tau_2 + \sigma_2 X_2 - t_3^*))],$$

which in special cases (I) and (II) simplifies to

$$\mathrm{VTTV}_2 = (\beta_3 + \gamma_3) \cdot E\left[\int_{\frac{\gamma_3 - \gamma_2^k}{\beta_3+\gamma_3}}^1 \Phi_{X_2 \mid X_1}^{-1}(p) dp\right].$$

Note the way that the dependence between travel times affects VTTV_2: The integral captures the mean lateness in standardized travel time (Fosgerau and Karlström, 2010), and when travel times are dependent, this also involves the mean across all realizations of X_1. If we define the expected inverse distribution function $\bar{\Phi}_{X_2}^{-1}$ as $\bar{\Phi}_{X_2}^{-1}(p) = E[\Phi_{X_2 \mid X_1}^{-1}(p)]$, $p \in [0,1]$, we can write VTTV_2 as

$$\mathrm{VTTV}_2 = (\beta_3 + \gamma_3) \int_{\frac{\gamma_3 - \gamma_2^k}{\beta_3+\gamma_3}}^1 \bar{\Phi}_{X_2}^{-1}(p) dp,$$

which is on the same form as in the single-trip step model. In general, $\bar{\Phi}_{X_2}^{-1}$ is different from the marginal inverse distribution function $\Phi_{X_2}^{-1}$. When travel times are independent, however, we have $\bar{\Phi}_{X_2}^{-1} = \Phi_{X_2}^{-1}$. When dependence is strong, we have $\Phi_{X_2 \mid X_1}^{-1} \approx X_1$, and since $E[X_1] = 0$, VTTV_2 tends to 0.

[**Figure 5 about here.**]

Figure 5: Illustration of the two-trip model with linear marginal costs.

5 Special case: Two trip slope model

We now consider another special case of the general model in Section 3 with linear marginal cost functions. This specification was previously studied in a setting of deterministic travel times by Jenelius et al. (2011), and is an extension of the single-trip model proposed by Fosgerau and Engelson (2011) studied in Section 2.4. The marginal cost functions are given by

$$\begin{aligned} c_1(t) &= \beta_0 + \beta_1 t, \\ c_2(t) &= \begin{cases} \gamma_0 + \gamma_1 t & t < \underline{t}, \\ \delta_0 + \delta_1 t & t \geq \bar{t}, \end{cases} \\ c_3(t) &= \epsilon_0 + \epsilon_1 t, \quad t \in [t_s, t_e], \end{aligned}$$

where $t_s < \underline{t} \leq \bar{t} < t_e$. We assume that the traveler always arrives during the first regime and always departs during the second regime of activity 2, respectively, i.e., that $(1-\xi)a_1 < \underline{t}$ and $s_2 - \xi a_1 > \bar{t}$ for all feasible arrival times a_1 and departure times s_2. The requirement that the traveler initially prefers the origin activity and ultimately the destination activity implies that $\beta_1 < \gamma_1$ and $\delta_1 < \epsilon_1$. We assume no particular sign for any of these parameters, although we will mainly discuss the case $\beta_1 < 0$, $\gamma_1 > 0$, $\delta_1 < 0$ and $\epsilon_1 > 0$, which complies with our general sufficient conditions for an interior optimum and may be the most common case in practice. With these parameter signs activities 1 and 2 end with a cool-down period with diminishing marginal utility, while activities 2 and 3 begin with a warm-up period with increasing marginal utility. As we will see, the traveler is then risk-averse on both trips.

We denote the optimal transition times between the activities when $T_1 = T_2 = 0$ as t_1^* and t_2^*, respectively. The first-order requirements $c_1(t_1^*) = (1-\xi)c_2((1-\xi)t_1^*) + \xi c_2(t_2^* - \xi t_1^*)$ and $c_2(t_2^* - \xi t_1^*) = c_3(t_2^*)$ allow us to express t_1^* and t_2^* in terms of the parameters of the marginal utility functions,

$$t_1^* = \frac{\beta_0 - \tilde{\gamma}_0}{\tilde{\gamma}_1 - \beta_1}, \quad t_2^* = \frac{\hat{\delta}_0 - \epsilon_0}{\epsilon_1 - \hat{\delta}_1}, \tag{16}$$

where we have introduced the useful auxiliary variables

$$\begin{aligned} \tilde{\gamma}_0 &\equiv (1-\xi)\gamma_0 + \xi \frac{\delta_0\epsilon_1 - \epsilon_0\delta_1}{\epsilon_1 - \delta_1}, \\ \tilde{\gamma}_1 &\equiv (1-\xi)^2\gamma_1 - \xi^2 \frac{\delta_1\epsilon_1}{\epsilon_1 - \delta_1}, \\ \hat{\delta}_0 &\equiv \frac{\delta_0[(1-\xi)^2\gamma_1 - \beta_1] - \xi[\beta_0 - (1-\xi)\gamma_0]\delta_1}{(1-\xi)^2\gamma_1 - \xi^2\delta_1 - \beta_1}, \\ \hat{\delta}_1 &\equiv \frac{[(1-\xi)^2\gamma_1 - \beta_1]\delta_1}{(1-\xi)^2\gamma_1 - \xi^2\delta_1 - \beta_1}. \end{aligned} \tag{17}$$

Note that with fixed scheduling, $\xi = 0$, we obtain $\tilde{\gamma}_0 = \gamma_0$, $\tilde{\gamma}_1 = \gamma_1$, $\hat{\delta}_0 = \delta_0$ and $\hat{\delta}_1 = \delta_1$. With completely flexible scheduling, $\xi = 1$, we obtain $\tilde{\gamma}_0 =$

$(\delta_0\epsilon_1 - \epsilon_0\delta_1)/(\epsilon_1 - \delta_1)$, $\tilde{\gamma}_1 = \delta_1\epsilon_1/(\epsilon_1 - \delta_1)$, $\hat{\delta}_0 = (\beta_0\delta_1 + \delta_0\beta_1)/(\beta_1 + \delta_1)$ and $\hat{\delta}_1 = \beta_1\delta_1/(\beta_1+\delta_1)$. With expected parameter signs $\beta_1 < 0 < \gamma_1$ and $\delta_1 < 0 < \epsilon_1$ we have $\tilde{\gamma}_1 > 0$ and $\hat{\delta}_1 < 0$ for all ξ.

5.1 Two trip slope model: optimality conditions

Since the scheduling of the last activity is fixed, the backward optimal marginal cost function defined in (7) is $\tilde{c}_3(a_2, (x_1, x_2)) = \epsilon_0 + \epsilon_1 a_2$. Applied to trip 2, the necessary condition (9) for the departure times gives

$$\delta_0 + \delta_1(s_2^* - \xi a_1) = E[\epsilon_0 + \epsilon_1(s_2^* + \tau_2 + \sigma_2 X_2 \,|\, x_1)],$$

which allows us to explicitly solve for the optimal departure time $s_2^* = s_2^*(a_1, x_1)$ as a function of the arrival time a_1 and the realized standardized travel time x_1 on trip 1,

$$s_2^*(a_1, x_1) = \frac{\delta_0 - \epsilon_0}{\epsilon_1 - \delta_1} - \xi\frac{\delta_1}{\epsilon_1 - \delta_1}a_1 - \frac{\epsilon_1}{\epsilon_1 - \delta_1}(\tau_2 + \sigma_2 E[X_2 \,|\, x_1]). \tag{18}$$

Note how the departure time s_2^* depends linearly on the arrival time to the preceding activity (unless $\xi = 0$) and the expected travel time on trip 2 conditional on the realized travel time on trip 1. An increase in the travel time mean or standard deviation on trip 2 moves the departure time earlier while a later arrival time moves the departure time later, if $\delta_1 < 0 < \epsilon_1$ as expected.

Consider then trip 1. The scheduling of the destination activity is flexible, and the backward optimal marginal cost function defined in (7), with the optimal departure time of trip 2 from (18) inserted, is

$$\begin{aligned}\tilde{c}_2(a_1, x_1) &= (1-\xi)(\gamma_0 + (1-\xi)\gamma_1 a_1) + \xi(\delta_0 + \delta_1(s_2^*(a_1, x_1) - \xi a_1)) \\ &= \tilde{\gamma}_0 + \tilde{\gamma}_1 a_1 - \xi\frac{\delta_1\epsilon_1}{\epsilon_1 - \delta_1}(\tau_2 + \sigma_2 E[X_2 \,|\, x_1]).\end{aligned}$$

The necessary optimality condition (9) applied to the timing of trip 1 now gives

$$\begin{aligned}\beta_0 + \beta_1 s_1^* &= E\left[\tilde{\gamma}_0 + \tilde{\gamma}_1 \cdot (s_1^* + \tau_1 + \sigma_1 X_1) - \xi\frac{\delta_1\epsilon_1}{\epsilon_1 - \delta_1}(\tau_2 + \sigma_2 E[X_2 \,|\, X_1])\right] \\ &= \tilde{\gamma}_0 + \tilde{\gamma}_1(s_1^* + \tau_1) - \xi\frac{\delta_1\epsilon_1}{\epsilon_1 - \delta_1}\tau_2.\end{aligned}$$

We can now solve explicitly for the optimal departure time on trip 1 and the associated arrival time to activity 2,

$$\begin{aligned}s_1^* &= t_1^* - \frac{\tilde{\gamma}_1}{\tilde{\gamma}_1 - \beta_1}\tau_1 + \xi\frac{\delta_1\epsilon_1}{(\tilde{\gamma}_1 - \beta_1)(\epsilon_1 - \delta_1)}\tau_2, \\ a_1^*(x_1) &= t_1^* - \frac{\beta_1}{\tilde{\gamma}_1 - \beta_1}\tau_1 + \xi\frac{\delta_1\epsilon_1}{(\tilde{\gamma}_1 - \beta_1)(\epsilon_1 - \delta_1)}\tau_2 + \sigma_1 x_1.\end{aligned} \tag{19}$$

We see that the optimal departure time depends linearly on the mean travel times on both trip 1 and (with $\xi > 0$) trip 2. With expected parameter signs $\beta_1 < 0 < \gamma_1$ and $\delta_1 < 0 < \epsilon_1$, an increase in the mean travel time on any trip moves the departure time earlier. Insertion of (19) into (18) gives the overall optimal departure time on trip 2 and associated arrival time as

$$\begin{aligned}
s_2^*(x_1) &= t_2^* + \xi \frac{\beta_1\delta_1}{(\tilde{\gamma}_1 - \beta_1)(\epsilon_1 - \delta_1)}\tau_1 - \frac{\epsilon_1}{\epsilon_1 - \hat{\delta}_1}\tau_2 \\
&\quad - \xi \frac{\delta_1}{\epsilon_1 - \delta_1}\sigma_1 x_1 - \frac{\epsilon_1}{\epsilon_1 - \delta_1}\sigma_2 E[X_2 \,|\, x_1], \\
a_2^*(x_1, x_2) &= t_2^* + \xi \frac{\beta_1\delta_1}{(\tilde{\gamma}_1 - \beta_1)(\epsilon_1 - \delta_1)}\tau_1 - \frac{\hat{\delta}_1}{\epsilon_1 - \hat{\delta}_1}\tau_2 \\
&\quad - \xi \frac{\delta_1}{\epsilon_1 - \delta_1}\sigma_1 x_1 - \frac{\epsilon_1}{\epsilon_1 - \delta_1}\sigma_2 E[X_2 \,|\, x_1] + \sigma_2 x_2,
\end{aligned} \tag{20}$$

which are linear in the travel time means as well as the standard deviations. With expected parameter signs, an increase in the mean travel time or standard deviation on trip 1 moves the departure time later, while a corresponding increase on trip 2 moves the departure time earlier.

5.2 Two trip slope model: value of mean travel time and variability

According to formula (10) and the optimal arrival time from (19), the unconditional value of mean travel time on trip 1 is

$$\begin{aligned}
\text{VMTT}_1 &= E\left[\tilde{\gamma}_0 + \tilde{\gamma}_1 \cdot a_1^*(X_1) - \xi \frac{\delta_1\epsilon_1}{\epsilon_1 - \delta_1}(\tau_2 + \sigma_2 E[X_2 \,|\, X_1])\right] \\
&= \frac{\beta_0\tilde{\gamma}_1 - \tilde{\gamma}_0\beta_1}{\tilde{\gamma}_1 - \beta_1} - \frac{\beta_1\tilde{\gamma}_1}{\tilde{\gamma}_1 - \beta_1}\tau_1 + \xi \frac{\beta_1\delta_1\epsilon_1}{(\tilde{\gamma}_1 - \beta_1)(\epsilon_1 - \delta_1)}\tau_2,
\end{aligned} \tag{21}$$

which is linearly increasing in the mean travel times of both trips but independent of the standard deviations and other characteristics of the joint travel time distribution. If the scheduling of activity 2 is completely fixed, i.e., $\xi = 0$, we have $\text{VMTT}_1 = (\beta_0\gamma_1 - \gamma_0\beta_1 - \beta_1\gamma_1\tau_1)/(\gamma_1 - \beta_1)$, independent of trip 2. As expected, this is identical to the value for the single-trip slope model considered in Section 2.4, given that the optimal transition time t_1^* is normalized to 0 (see also Fosgerau and Engelson, 2011). If the scheduling is completely flexible, $\xi = 1$, we have $\text{VMTT}_1 = (\epsilon_0\beta_1\delta_1 - \delta_0\beta_1\epsilon_1 - \beta_0\delta_1\epsilon_1 + \beta_1\delta_1\epsilon_1(\tau_1 + \tau_2))/(\beta_1\delta_1 - \delta_1\epsilon_1 - \beta_1\epsilon_1)$, which is symmetric in the two mean travel times.

Further, according to formula (10) and the optimal arrival time from (20), the unconditional value of mean travel time on trip 2 is

$$\begin{aligned}
\text{VMTT}_2 &= E[\epsilon_0 + \epsilon_1 \cdot a_2^*(X_1, X_2)] \\
&= \frac{\hat{\delta}_0\epsilon_1 - \epsilon_0\hat{\delta}_1}{\epsilon_1 - \hat{\delta}_1} + \xi \frac{\beta_1\delta_1\epsilon_1}{(\tilde{\gamma}_1 - \beta_1)(\epsilon_1 - \delta_1)}\tau_1 - \frac{\hat{\delta}_1\epsilon_1}{\epsilon_1 - \hat{\delta}_1}\tau_2,
\end{aligned} \tag{22}$$

which, again, is linearly increasing in the mean travel times of both trips. If the scheduling of activity 2 is completely fixed, i.e., $\xi = 0$, we have $\text{VMTT}_2 = (\delta_0\epsilon_1 - \epsilon_0\delta_1 - \delta_1\epsilon_1\tau_2)/(\epsilon_1 - \delta_1)$, independent of trip 1. Again, this is equivalent to the single-trip slope model given that the optimal transition time t_2^* is normalized to 0. If the scheduling is completely flexible, $\xi = 1$, we have $\text{VMTT}_2 = (\epsilon_0\beta_1\delta_1 -$

$\delta_0\beta_1\epsilon_1 - \beta_0\delta_1\epsilon_1 + \beta_1\delta_1\epsilon_1(\tau_1 + \tau_2))/(\beta_1\delta_1 - \delta_1\epsilon_1 - \beta_1\epsilon_1)$, which is identical to VMTT_1 for the same case.

Thus, with flexible scheduling there is a single unconditional value of mean travel time for both trips. This result is not true for general marginal cost functions $c_i(t)$. Specifically, it stems from the fact that for both trips and any level of flexibility ξ, the VMTT in this linear marginal cost specification is independent of the travel time variability. This means that it is equal to the marginal value of travel time (VTT) for the case with deterministic travel times, which is derived by Jenelius et al. (2011). It is shown in that paper that the VTT for the two trips are equal when $\xi = 1$ and travel times are independent of departure times; in fact, this latter result holds not only for linear marginal costs but for general functions $c_i(t)$.

According to formula (12), the unconditional value of travel time variability on trip 1 is

$$\begin{aligned}\text{VTTV}_1 &= E\left[X_1 \cdot \left(\tilde{\gamma}_0 + \tilde{\gamma}_1 \cdot a_1^*(X_1) - \xi\frac{\delta_1\epsilon_1}{\epsilon_1 - \delta_1}(\tau_2 + \sigma_2 E[X_2 \mid X_1])\right)\right] \\ &= \tilde{\gamma}_1\sigma_1 - \xi\frac{\delta_1\epsilon_1}{\epsilon_1 - \delta_1}\sigma_2 \text{Cov}[X_1, X_2],\end{aligned}$$

which is independent of the travel time means and linear in the travel time standard deviation of both trip 1 and (if $\xi > 0$) trip 2. The first term captures the direct impact of the variability of the arrival time to activity 2, incorporating the endogenous choice of departure time on trip 2 through the proportionality constant $\tilde{\gamma}_1$. The second term is an addition compared to the single-trip model in Section 2.4 and captures the amount that the realized travel time on trip 1 affects the expectation about the travel time on trip 2, and hence the departure time on the trip. This dependency enters the VTTV in the form of the covariance between the standardized travel times. We would typically expect the covariance to be positive if different from zero, which means that it contributes to a higher cost of travel time variability if $\delta_1 < 0 < \epsilon_1$ as expected. The term vanishes when $\xi = 0$.

Similarly, the unconditional value of travel time variability on trip 2 is, applying formula (12),

$$\begin{aligned}\text{VTTV}_2 &= E[X_2 \cdot (\epsilon_0 + \epsilon_1 \cdot a_2^*(X_1, X_2))] \\ &= \epsilon_1\sigma_2 - \xi\frac{\delta_1\epsilon_1}{\epsilon_1 - \delta_1}\sigma_1 \text{Cov}[X_1, X_2] - \frac{\epsilon_1^2}{\epsilon_1 - \delta_1}\sigma_2 \text{Var}[E[X_2 \mid X_1]].\end{aligned}$$

Again, the first term captures the direct impact of the variability of the arrival time to activity 3. The correlation between the travel times affects the scheduling of trip 1 through the departure time on trip 2 and enters the cost symmetrically to trip 1, with σ_1 replacing σ_2 in the second term. The third, non-positive term captures the increased ability to predict the travel time on trip 2 having observed the travel time on trip 1. This effect enters the VTTV in the form of the variance of the expectation of X_2 conditional on X_1, $\text{Var}[E[X_2 \mid X_1]]$, or loosely speaking how much the expectation about the travel time on trip 2 varies depending on the realized travel time on trip 1. With $\epsilon_1 \neq 0$ this represents a reduction of the cost that does not depend on the scheduling flexibility of activity 2.

[Table 1 about here.]

Table 1: Parameter values for the linear marginal cost functions. The flexibility parameter for activity 2 is set to $\xi = 0.2$.

Note that if X_1 and X_2 are independent, then $\text{Cov}[X_1, X_2] = 0$ and $\text{Var}[E[X_2 \mid X_1]] = \text{Var}[E[X_2]] = \text{Var}[0] = 0$, so that all interaction terms in the VTTV disappear. We may also have $\text{Cov}[X_1, X_2] = 0$ or $\text{Var}[E[X_2 \mid X_1]] = 0$ even though X_1 and X_2 are not independent. Furthermore, $\text{Var}[E[X_2 \mid X_1]] = 0$ implies $\text{Cov}[X_1, X_2] = 0$, while the opposite is not true in general. In practice, however, it seems likely that dependence of travel times would most often manifest itself as correlation.

5.3 Numerical example

In this section we study some properies of the model numerically for a realistic set of parameter values, although the results should be seen mainly as an illustration of the model to be followed by more rigorous estimation and more reliable results in future work. We are mainly interested in the impact of travel time correlation on the VMTT and VTTV on each trip.

The parameters are calibrated so that the linear marginal cost functions correspond as closely as possible to the marginal cost functions used in Jenelius et al. (2011) to model a day consisting of three activities: being at home in the morning, working during the day and being at home in the evening. In that paper, sigmoid logistic marginal cost functions are calibrated to match the empirical results presented in Tseng and Verhoef (2008); we refer to the former paper for a detailed description of the procedure. In this paper we calibrate the slope parameters β_1, γ_1, δ_1 and ϵ_1 to be equal to the slopes of the corresponding logistic functions of Jenelius et al. (2011) at their saturation (or "mid") points. The values are reported in Table 1 and fulfil our expectations about the parameter signs from the analysis above.

As in Jenelius et al. (2011), the flexibility parameter is set to $\xi = 0.2$. The intercept parameters β_0, γ_0, δ_0 and ϵ_0 are calibrated so that the constant term of the VMTT on each trip (i.e., the VMTT in the limit $\tau_1 = \tau_2 = 0$) is -2.0 Euro/h, in line with Tseng and Verhoef (2008).[3] While it has no direct impact on the time values, they are further calibrated to give the optimal transition times $t_1^* = 8.0$ h and $t_2^* = 17.0$ h, which may represent a typical work day. The parameter values that uniquely correspond to these conditions can be solved for using formulas (16), (17), (21) and (22) and are shown in Table 1.

With the calibrated parameter values, the value of mean travel time for each trip is $\text{VMTT}_1 = -2.0 + 11.6\tau_1 + 0.79\tau_2$ Euro/h and $\text{VMTT}_2 = -2.0 + 0.79\tau_1 + 10.9\tau_2$ Euro/h. The fact that VMTT_1 depends more on τ_1 than on τ_2 and vice versa is due to the relatively low scheduling flexibility. Further, the value of travel time variability on each trip is $\text{VTTV}_1 = 32.4\sigma_1 + 2.20\rho\sigma_2$ Euro/h and $\text{VTTV}_2 = 2.20\rho\sigma_1 + (19.6 - 8.64\rho^2)\sigma_2$ Euro/h. Thus, while VTTV_1 increases linearly with the travel time correlation ρ, VTTV_2 follows a second-degree polynomial in ρ, obtaining the maximum value at $\rho = 0.13\sigma_1/\sigma_2$ if this

[3] The finding that VMTT < 0 for short trips may be due to estimation errors but may also capture a real positive value associated with making short trips (Tseng and Verhoef, 2008).

[**Figure 6 about here.**]

Figure 6: The unconditional VTTV on trip 1 (top) and trip 2 (bottom) as functions of the travel time correlation ρ for different values of the travel time standard deviation on trip 1 (left) and on trip 2 (right). To the left, σ_1 increases linearly from 0 to 20 minutes while σ_2 is constant at 10 minutes. To the right, symmetrically, σ_2 increases linearly from 0 to 20 minutes while σ_1 is constant at 10 minutes. Parameter values are given in Table 1.

is less than 1. For higher values of ρ, VTTV_2 decreases as the positive effect of correlation (accurate travel time prediction) overcomes the negative effect (the exacerbation of delay impacts). Figure 6 shows how the VTTV on each trip varies with the travel time correlation ρ for different values of σ_1 and σ_2, respectively, holding the other variable constant.

While the purpose here is not to estimate the VMTT and VTTV, some comments regarding the practical relevance of the numerical results may be in place. Considering that there is a significant time between the two trips, we would expect the travel time correlation to be rather weak in practice. Further, the calculated VMTT are significantly lower than those found in most empirical studies (see, e.g., Abrantes and Wardman, 2011) for typical travel times. They are, however, in line with the results of Tseng and Verhoef (2008) against which the parameters were calibrated. It follows that the ratio VTTV/VMTT is considerably larger than what is commonly reported (e.g., Noland and Polak, 2002).

6 Conclusion and discussion

The main contribution of this paper is to extend the analysis of the value of mean travel time (VMTT) and travel time variability (VTTV) from simple single-trip scheduling models to a more general multi-trip setting, incorporating the effects of flexibility in activity scheduling and dependence of travel times across trips. We also give sufficient conditions under which this generalization of the analysis approach is possible. As a first step, we reviewed a unifying single-trip model, of which current popular models are special cases, and derived formulas for the VMTT and VTTV. These are based directly on the marginal cost functions of the departure and arrival times, which avoids the need to explicitly calculate the expected cost function. The formulas serve as a platform from which specific models with desirable properties or empirical support can be obtained, both when closed form expressions exist and when numerical calculations are necessary.

The multi-trip model incorporates two features that are not captured by single-trip models: First, the scheduling of activities may be flexible, meaning that the utility derived from them depends not only on the time of day but also on the duration of activity participation. Second, travel times on different trips may be correlated, which is particularly relevant if they are close in time and space. The analysis shows that these features affect the VMTT and VTTV on a trip separately as well as in interaction with each other: Scheduling flexibility means that the VMTT and VTTV depend on the preceding arrival time and hence on the travel times on preceding trips, whether these are statistically dependent or not. Travel time dependence means that preceding realized travel times provide information about future travel times and hence affect scheduling and the VMTT and VTTV, whether scheduling is flexible or not.

We have considered two specific form of the multi-trip model with two trips and linear marginal cost functions that are piecewise constant in the first case and linear in the other case, and closed-form expressions for the VMTT and VTTV are obtained. The case with linear marginal costs is particularly rich on insights. In this case the formulas show that the dependence of the travel times affects the VTTV in two ways. First, if the scheduling of the intermediate activity is somewhat flexible, there are (with expected parameter signs) negative impacts on both trips associated with a positive correlation of travel times since it decreases the time spent in the activity. Second, there is a benefit in the dependence for trip 2, since it allows a more accurate expectation about the travel time on trip 2 on a given day (i.e., given the realized travel time on trip 1), and hence a more efficient scheduling of the trip.

The analysis of this paper opens up new dimensions to the value of reducing travel time variability. We can classify disturbances according to their persistence over time and the number of trips that are affected, so that their travel times are correlated, and we can assess the value of reducing persistent variability as opposed to transient, trip-specific variability. This is particularly relevant for transport modes where persistent disturbances sometimes are common, for example railway and airway traffic.

Another task for future research is to test the predictions of the multi-trip model, such as choices of departure times, against empirical data. In order to apply the model in practice, the scheduling flexibility of activities and the correlation of trip travel times need to be estimated, which requires more information than single trip-models. Another line of empirical work is to investigate the dependence between trip travel times throughout a day.

The only decision variables in the present model are the departure times on the trips. In future work it would be valuable to incorporate other choice dimensions such as trip cancelling and route, destination and mode changes. It would also be relevant to consider dependencies between activities (for example, the cancelling of an activity could make a subsequent activity less attractive) and between days (for example, an activity could be postponed to the following day, although with a larger cost of further postponement). Simply put, our directions for further development are much the same as within the field of activity-based modeling in general. Although the rapidly increasing number of decision variables would make the analysis increasingly challenging, the essential features would remain the same. In particular, the dynamic programming approach and the concept of the backward optimal marginal cost function, properly generalized, should still be relevant for calculating the VMTT and VTTV.

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Appendix

We show here that the backward optimal marginal cost function is increasing and continuously differentiable under the sufficient conditions in Section 3.4. To repeat, we defined the utility function

$$\tilde{U}_n(s_n, a_{n-1}, \mathbf{x}_{0:n-1}) \equiv \int_{(1-\xi_n)a_{n-1}}^{s_n - \xi_n a_{n-1}} c_n(t)dt + E\left[\int_{a_n(s_n, X_n)}^{t_e} c_{n+1}(t)dt \,\middle|\, \mathbf{x}_{0:n-1}\right],$$

and for each preceding activity $i \in \{1, \ldots, n-1\}$ the recursive utility function

$$\begin{aligned} \tilde{U}_i(s_i, a_{i-1}, \mathbf{x}_{0:i-1}) \equiv & \int_{(1-\xi_i)a_{i-1}}^{s_i - \xi_i a_{i-1}} c_i(t)dt \\ & + E\big[\tilde{U}_{i+1}(s^*_{i+1}, a_i(s_i, X_i), \mathbf{x}_{0:i-1}, X_i) \,\big|\, \mathbf{x}_{0:i-1}\big]. \end{aligned}$$

Further, we defined the backward optimal marginal cost function as $\tilde{c}_{n+1}(a_n, \mathbf{x}) = c_{n+1}(a_n)$, and for each activity $i \in \{2, \ldots, n\}$,

$$\begin{aligned} \tilde{c}_i(a_{i-1}, \mathbf{x}_{0:i-1}) &\equiv -\frac{d}{da_{i-1}} \tilde{U}_i(s^*_i, a_{i-1}, \mathbf{x}_{0:i-1}) \\ &= (1-\xi_i) \cdot c_i((1-\xi_i) \cdot a_{i-1}) + \xi_i \cdot c_i(s^*_i(a_{i-1}, \mathbf{x}_{0:i-1}) - \xi_i a_{i-1}). \end{aligned}$$

Through induction we show that $\tilde{c}_i(a_{i-1}, \mathbf{x}_{0:i-1})$ is an increasing function of a_{i-1} for all $\mathbf{x}_{0:i-1}$ and $i \in \{2, \ldots, n+1\}$ under the sufficient conditions in Section 3.4. The sufficient conditions state that $\tilde{c}_{n+1}(a_n, \mathbf{x}) = c_{n+1}(a_n)$ is increasing. Consider an arbitrary activity $i \in \{2, \ldots, n\}$ and assume that the subsequent backward optimal marginal cost function $\tilde{c}_{i+1}(a_i, \mathbf{x}_{0:i})$ is increasing for all $\mathbf{x}_{0:i}$. The derivative of $\tilde{c}_i(a_{i-1}, \mathbf{x}_{0:i-1})$ with respect to the arrival time a_{i-1} is

$$\begin{aligned} \tilde{c}'_i(a_{i-1}, \mathbf{x}_{0:i-1}) &= -\frac{d^2}{da^2_{i-1}} \tilde{U}_i(s^*_i, a_{i-1}, \mathbf{x}_{0:i-1}) \\ &= (1-\xi_i)^2 \cdot c'_i((1-\xi_i) \cdot a_{i-1}) \\ &\quad + \xi_i \cdot \left(\frac{ds^*_i}{da_{i-1}} - \xi_i\right) \cdot c'_i(s^*_n(a_{i-1}, \mathbf{x}_{0:i-1}) - \xi_i a_{i-1}). \end{aligned} \tag{23}$$

In order for $\tilde{c}_i$ to be increasing, $\tilde{c}'_i$ must be positive. Under the sufficient conditions, $c'_i((1-\xi_i)a_{i-1})$ is positive and $c'_i(s^*_n(a_{i-1}, \mathbf{x}_{0:i-1}) - \xi_i a_{i-1})$ is negative for all feasible a_{i-1}. It remains to determine ds^*_i/da_{i-1}, i.e., the marginal change in departure time induced by a change in preceding arrival time. By the rules for implicit differentiation,

$$\frac{ds^*_i}{da_{i-1}} = -\frac{\partial^2 \tilde{U}_i}{\partial a_{i-1} \partial s_i} \Bigg/ \frac{\partial^2 \tilde{U}_i}{\partial s_i^2},$$

evaluated at $(a_{i-1}, s^*_i(a_{i-1}, \mathbf{x}_{0:i-1}))$, where

$$\begin{aligned} \frac{\partial^2 \tilde{U}_i}{\partial a_{i-1} \partial s_i} &= -\xi_i c'_i(s_i - \xi_i a_{i-1}), \\ \frac{\partial^2 \tilde{U}_i}{\partial s_i^2} &= c'_i(s_i - \xi_i a_{i-1}) - E\big[a''_i(s_i, X_i) \cdot \tilde{c}_{i+1}(a_i(s_i, X_i), \mathbf{x}_{0:i-1}, X_i) \\ &\qquad + a'_i(s_i, X_i) \cdot \tilde{c}'_{i+1}(a_i(s_i, X_i), \mathbf{x}_{0:i-1}, X_i) \,\big|\, \mathbf{x}_{0:i-1}\big]. \end{aligned} \tag{24}$$

Under the sufficient conditions, travel time is independent of departure time, which means that $a_i'(s_i, X_i) \equiv 1$ and $a_i''(s_i, X_i) \equiv 0$. In this case, $\partial^2 \tilde{U}_i / \partial s_i^2$ simplifies to

$$\frac{\partial^2 \tilde{U}_i}{\partial s_i^2} = c_i'(s_i - \xi_i a_{i-1}) - E\big[\tilde{c}_{i+1}'(a_i(s_i, X_i), \mathbf{x}_{0:i-1}, X_i) \,\big|\, \mathbf{x}_{0:i-1}\big], \tag{25}$$

Since $\tilde{c}_{i+1}(a_i, \mathbf{x}_{0:i})$ is increasing in a_i for all $\mathbf{x}_{0:i}$ by assumption, the derivative $\tilde{c}_{i+1}'(a_i, \mathbf{x}_{0:i})$ is positive. The expectation across all realizations of X_i is thus positive as well. We thus have

$$\frac{ds_i^*}{da_{i-1}} = \frac{\xi_i c_i'(s_i^* - \xi_i a_{i-1})}{c_i'(s_i^* - \xi_i a_{i-1}) - E\big[\tilde{c}_{i+1}'(a_i(s_i^*, X_i), \mathbf{x}_{0:i-1}, X_i) \,\big|\, \mathbf{x}_{0:i-1}\big]}, \tag{26}$$

which lies in the interval $[0, \xi_i]$ for any ξ_i. The change in departure time is thus positive but smaller than the change in preceding arrival time. Inserting (26) into (23) gives that $\tilde{c}_i'(a_{i-1}, \mathbf{x}_{0:i-1}) \geq 0$ at least up to the peak of c_i at $(1 - \xi_i)a_{i-1} = \hat{t}_i$; hence $\tilde{c}_i(a_{i-1}, \mathbf{x}_{0:i-1})$ is increasing. It follows by induction that $\tilde{c}_i$ is increasing for all $i \in \{2, n+1\}$.

Note that the condition that travel time is independent of departure time can be relaxed as long as the expected value in (24) is still positive, which ensures that $ds_i^*/da_{i-1} \leq \xi_i$ and that $\tilde{c}_i$ is increasing.

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Figure captions

Figure 1: Illustration of the single-trip model and the optimal departure time. The curved, dashed line at time a indicates that the arrival time is stochastic.

Figure 2: Illustration of a trip i in the multiple-trip model and the optimal departure time s_i^*. The double, dashed lines for $\tilde{c}_{i+1}(t, \mathbf{x}_{0:i-1}, X_i)$ indicates that the backward optimal marginal cost function is stochastic. The curved, dashed line at time a_i indicates that the arrival time is stochastic.

Figure 3: Illustration of the sufficient conditions for the existence of an interior solution, which must hold simultaneously for all trips.

Figure 4: Illustration of the two-trip model with piecewise constant marginal costs.

Figure 5: Illustration of the two-trip model with linear marginal costs.

Figure 6: The unconditional VTTV on trip 1 (top) and trip 2 (bottom) as functions of the travel time correlation ρ for different values of the travel time standard deviation on trip 1 (left) and on trip 2 (right). To the left, σ_1 increases linearly from 0 to 20 minutes while σ_2 is constant at 10 minutes. To the right, symmetrically, σ_2 increases linearly from 0 to 20 minutes while σ_1 is constant at 10 minutes. Parameter values are given in Table 1.

Table captions

Table 1: Parameter values for the linear marginal cost functions. The flexibility parameter for activity 2 is set to $\xi = 0.2$.

Tables

Table 1:

Marginal cost function	Intercept (Euro/h)	Slope (Euro/h^2)
Morning home: $c_1(t) = \beta_0 + \beta_1 t$	$\beta_0 = 143$	$\beta_1 = -18.1$
Morning work: $c_2(t) = \gamma_0 + \gamma_1 t, \quad t < \underline{t}$	$\gamma_0 = -322$	$\gamma_1 = 50.0$
Afternoon work: $c_2(t) = \delta_0 + \delta_1 t, \quad t \geq \bar{t}$	$\delta_0 = 383$	$\delta_1 = -25.0$
Evening home: $c_3(t) = \epsilon_0 + \epsilon_1 t$	$\epsilon_0 = -335$	$\epsilon_1 = 19.6$