

**Road network vulnerability analysis of area-covering disruptions:
A grid-based approach with case study**

Erik Jenelius (corresponding author)
erik.jenelius@abe.kth.se
Phone: +46 8 790 83 02
Fax: +46 8 790 70 02

Lars-Göran Mattsson
lars-goran.mattsson@abe.kth.se

Department of Transport Science,
Royal Institute of Technology (KTH),
Teknikringen 10, SE-100 44 Stockholm, Sweden

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Abstract

We present an approach to systematically analyzing the vulnerability of road networks under disruptions covering extended areas. Since various kinds of events including floods, heavy snowfall, storms and wildfires can cause such spatially spread degradations, the analysis method is an important complement to the existing studies of single link failures. The methodology involves covering the study area with grids of uniformly shaped and sized cells, where each cell represents the extent of an event disrupting any intersecting links. We apply the approach to the Swedish road network using travel demand and network data from the Swedish national transport modelling system Sampers. The study shows that the impacts of area-covering disruptions are largely determined by the level of internal, outbound and inbound travel demand of the affected area itself. This is unlike single link failures, where the link flow and the redundancy in the surrounding network determine the impacts. As a result, the vulnerability to spatially spread events shows a markedly different geographical distribution. These findings, which should be universal for most road networks of similar scale, are important in the planning process of resource allocation for mitigation and recovery.

Keywords: vulnerability, robustness, road network, transport, disruption, area

1. Introduction

1.1 Background

The road transport system is one of the fundamentals of modern society. Its ability to connect spatially separated locations is vital for the accessibility and welfare of people and the economic efficiency of businesses. As a result, unplanned degradations in the system, when they occur, often have severe consequences (e.g., Wesemann et al., 1996; Zhu et al. 2010). In the worst cases, such disruptions can threaten the possibility for some people to receive medical care and other critical services. More generally, they can lead to increases in travel time that impair the ability of individuals to take part in their daily activities, including commuting, dropping off and picking up children from day-care and school, doing the shopping, etc. For businesses, the impacts can include delayed deliveries and supplies, loss of customers and manpower, increased freight costs, etc.

Events of widely varying nature can cause degradations of the road network. Some events originate within the transport system, including car crashes and technical failures such as bridge collapses and incidents induced by road works. Events of this kind are typically concentrated to a single link in the network. Other events are external strains imposed on the system. These may be caused by man, as with industrial leakages and sabotage, or by nature, as with floods, landslides, heavy snowfall, storms, wildfires, earthquakes and other kinds of natural hazards. Since they originate outside of the transport system, events caused by nature may extend across larger areas, potentially disrupting several links.

Given the value of a functional road transport system, it is important to be able to predict the impacts of disruptions in the system, where they are the most likely to occur, and where the impacts would be the most severe. This can be said to form the basic goal of road network vulnerability analysis. Such an assessment may be followed by identifying feasible actions to prevent, mitigate or recover from future disturbances and the costs associated with these. This, in turn, would make it possible to manage the level of vulnerability in the system considering the costs and benefits of different actions and resource allocations.

Traditionally, transport policy and planning has been focused on the performance of the transport system under normal or average demand and supply conditions. In recent years it has been increasingly recognized that variations from the normal state can cause considerable reductions in efficiency. For example, the cost of travel time uncertainty (e.g., Bates et al., 2001) and the reliability of the transport system under fluctuations in capacity or demand (e.g., Clark and Watling, 2005) have been studied. The field of network vulnerability, mainly focusing on the impacts of significant network degradations with less emphasis on probabilities, has received growing attention as well. According to Berdica (2002), vulnerability is “a susceptibility to incidents that can result in considerable reductions in road network serviceability”. The serviceability of a link/route/road network, in turn, is defined as “the possibility to use that link/route/road network during a given time period”. This general notion of vulnerability will be adopted in the present paper.

Inspired by the typology of Murray et al. (2008), the road network vulnerability literature can be broadly classified with respect to the kind and range of scenarios that are studied. A first category of studies focuses on one or a few specific scenarios. Notably, several authors have assessed the economic impacts of earthquakes disrupting the road network using integrated transport network and multiregional trade models (Cho et al., 2001; Kim et al., 2002; Ham et

al., 2005; Tatano and Tsuchiya, 2009). Methods based on transport modelling systems are used by Suarez et al. (2005), Berdica and Mattsson (2007) and Taylor (2008) to study the impacts of network degradations in urban areas.

A second category of studies considers a full range of scenarios. So far, this category contains a number of complete studies of single link failures (Taylor et al., 2006; Jenelius et al., 2006; Sohn, 2006; Knoop et al., 2008; Erath et al., 2010). The different studies differ mainly in the metrics and models that are used to evaluate the impacts. In all studies, however, the increase in travel time or travel distance for the drivers is a vital component of the evaluation.

A third category of studies uses optimization and game-theoretical techniques to identify worst-case scenarios, or best responses to such scenarios (e.g. Bell et al., 2008; Matisziw and Murray, 2009). In addition to these applied studies, a number of authors have proposed methods, models and metrics to evaluate road network vulnerability and quantify component importance (Murray-Tuite and Mahmassani, 2004; Chen et al., 2007; Nagurney and Qiang, 2008; Sullivan et al., 2010). These models could potentially be used in any of the three approaches listed above.

As Murray et al. (2008) note, the different approaches all have their merits and shortcomings. With a full range approach, one gets a comprehensive picture of the vulnerability of the network, including worst-case scenarios as well as distributions across users and regions. It is also possible to draw general conclusions about the factors underlying vulnerability (Jenelius, 2009). This comes at the price of extensive calculations, which may require simplified models and vulnerability metrics to be feasible. Such simplifications, in turn, may mean that important features of the system and individual scenarios are overlooked. With a scenario-specific assessment, on the other hand, it is possible to use sophisticated analytical models that can represent the features and complexity of the particular scenario and system. In combination, a full range study may bring attention to certain scenarios and parts of the network, for which more in-depth studies then may be performed.

1.2 Aim of the paper

While several of the scenario-specific studies mention above have considered multiple link failures, full range vulnerability studies have in principle been limited to single link failures. Since many kinds of events can cover and disable extended areas of the road network, it is important to investigate to what extent the impacts differ compared to single link failures. It can be expected, for example, that the benefit of a dense network with many alternative routes is smaller, since there is a risk that many or all nearby links are simultaneously disrupted during the event. If the characteristics of the impacts are indeed different, it is necessary to consider both kinds of events with suitable methodologies in vulnerability analyses.

In this paper we present a full range methodology for studying road network vulnerability under area-covering disruptions. Our focus is on events where a substantial portion of the road network within the affected area is severely degraded. With our approach, the study area including the transport network model is covered by grids of uniformly shaped and sized cells, where each cell represents the spatial coverage of a disrupting event. For each cell, any links intersecting it are identified and the consequences of disabling these links simultaneously are calculated. Multiple grids of the same cell size, evenly displaced from each other, are used to increase the accuracy of the analysis. The approach allows us to systematically study the impacts of disrupting events depending on their spatial location and

extent. We can also identify users and geographical regions that are particularly exposed to events of this kind. The approach handles the potential combinatorial problems associated with multiple link failures and is not biased regarding what parts of the transport system are covered by the disruptions.

It should be noted that a similar method is used by Patterson and Apostolakis (2007) to identify critical locations where a single terrorist attack may damage multiple infrastructure systems. Also related, Kurauchi et al. (2009) consider the vulnerability of an origin-destination (OD) pair as the extent to which a link failure can reduce the number of non-overlapping routes between the OD pair. This approach acknowledges the risk of being dependent on a few critical links, but does not capture the fact that spatial proximity introduces dependencies between links that are not apparent from network topology alone.

The methodology is applied in a case study of the Swedish road transport system. We calculate the impacts, in terms of unsatisfied travel demand and delays for the users, of area-covering disruptions across the country using different cell sizes and closure durations. The impacts are compared with the impacts of single link failures in the corresponding areas in order to assess the differences and similarities between the two types of events. We also study worst-case scenarios at a regional level. The results are analysed considering the characteristics of the road network, travel patterns and location patterns in different parts of the study area, and general conclusions regarding the mechanisms behind the geographical variations in vulnerability are drawn.

The rest of the paper is structured as follows. The methodology is described in the Section 2, followed by a presentation of the case study in Section 3 and results given in Section 4. The paper ends with discussion and conclusions in Section 5.

2. Methodology

2.1 Grid-based vulnerability analysis

When extending full range road network vulnerability analysis from single link failures to area-covering disruptions, there are two issues that must be taken into account. The first is that there are in reality infinitely many possible locations and extents for a disruptive event, in contrast to the finite set of road links. In order to perform an analysis, we need to discretise and limit the number of possible events to consider. As we are interested in systematically studying how the impacts of disruptions differ between geographical regions, this should be done in a way that is not biased towards certain regions. This is particularly valuable if we want to assign probabilities or frequencies to the different events. For example, it might at first seem appropriate to approximate the extent of the disrupting event with a circular area, so that it does not matter how the area is rotated in relation to the study area. However, no matter how closely we position and overlap these circles, some parts of the study area will be covered by more circles, i.e. events, than others. This creates a bias in the results towards areas that happen to be more covered.

The second issue is the combinatorial problems associated with analyzing multiple link failures. In reality, an event affecting an extended area may cause some links to be completely blocked, others to have their capacities reduced, and still others to be virtually unaffected. The number of possible combinations of link states quickly makes it unfeasible to consider all

these combinations for realistic road network sizes. Hence, there is a need for a systematic method to define and limit the set of scenarios to analyze.

Our approach to handle these issues is to make a complete coverage of the study area using evenly displaced *grids* of uniformly shaped and sized cells. Each cell represents the precise spatial location and extent of a disrupting event. To simulate the event, any road links intersecting the cell (fully or partially) are completely closed for the duration of the disruption, while all links outside the cell are completely unaffected. The impact of the disruption is then calculated using the analyst's choice of models and metrics.

The grid-based approach guarantees that each point in the study area is covered by the same number of events. By superimposing multiple grids of the same cell size and shape, we can control the detail and accuracy of the analysis in relation to the computational burden. Using G grids, each containing R rows and C columns of cells, the maximum number of events that we need to consider is $G \times R \times C$; the actual number may be lower since we should disregard any cells that do not intersect the study area. By superimposing grids of different cell sizes, we can evaluate the vulnerability of the transport system under a range of spatial extents for the disruptions. The cells may be assigned weights to capture that the likelihood of different sizes of disruptions may vary across the study area.

The three kinds of grids that may be used are hexagonal, square and triangular grids. The hexagonal grids may be theoretically preferable since the cells are the most circular, so that the results depend less on how the grids are rotated relative to the study area. The square grids, on the other hand, may be more convenient to work with in practice. Figure 1 illustrates four square grids symmetrically displaced so that four cells cover every point in the study area.

Figure 1 about here.

2.2 Cell importance and regional exposure

In parallel with the concept of link importance (Jenelius et al., 2006; Nagurney and Qiang, 2008; Sullivan et al., 2010), we define the *importance* of a particular grid cell to be the total impact for the network users if all links intersecting the cell are closed for a certain duration.¹ Since the focus is on the overall reduction in efficiency of the transport system, the importance measure represents an infrastructure-centred, operator's perspective on road network vulnerability. Analyzing importance provides information on where to allocate resources for reducing the impacts of disrupting events the most effectively.

Formally, let $\Delta C_{ij}^c(\tau)$ be the total impact for all users travelling from origin i to destination j , where j may equal i if they represent aggregated zones, during a closure of all links intersecting cell c with duration τ . We then define the importance of cell c , given closure duration τ , as the total impact over all OD relations in the network,

$$I(c | \tau) = \sum_i \sum_j \Delta C_{ij}^c(\tau). \quad (1)$$

¹ Since the cells are components of the analysis method rather than the transport system, it would perhaps be more appropriate to talk of the collective importance of the links intersecting the cell rather than the importance of the cell itself. However, we will use the term cell importance for simplicity.

The concept of regional *exposure* was introduced in Jenelius et al. (2006). Given a disruption scenario, the exposure of a region is the impact for the users travelling from the region to anywhere in the network if the scenario would occur. Jenelius (2009) further divides exposure into total exposure, measured as the total impact across the users, and user exposure, measured as the average impact per user. In this paper we will focus on the latter measure. The user exposure measure represents a user's perspective on road network vulnerability, since it captures how individuals are affected by disruptive events depending on their location. It should also be of interest to the operating authorities who may have to take equity and regional development issues into account in their planning.

In this paper we consider a worst-case scenario, where the cell having the most severe impact for each region is closed. Formally, let r denote a region in the study area and let $i \in r$ mean that origin i is located within region r . Let x_{ij} denote the average travel demand (in trips or vehicles per unit time) between i and j during the closure. Given closure duration τ and a collection of grids \mathcal{G} , the worst-case user exposure of region r is then

$$E(r | \tau, \mathcal{G}) = \max_{c \in \mathcal{G}} \frac{\sum_{i \in r} \sum_j \Delta C_{ij}^c(\tau)}{\sum_{i \in r} \sum_j x_{ij} \tau}. \quad (2)$$

Note that the denominator, i.e., the total travel demand originating in the region during the closure duration, does not affect which cell has the most severe impact, but it may affect the relative exposure of different regions.

2.3 Assessing disruption impacts

The size of the grid cells and the model used to evaluate disruption impacts should be chosen jointly to capture the characteristics of the kind of events that is relevant for the analysis. Some hazards, such as accidental releases of toxic chemicals, may be quite limited in space and time, whereas others, such as earthquakes, may affect significant areas over a long period of time with severe and complex humanitarian and economic consequences. Just as in single link vulnerability analysis, any model and metrics appropriate for the purpose of the particular study and possible to apply with available data can be used for impact assessment in the grid-based approach. As noted in Section 1, full range vulnerability analyses require extensive calculations, something that limits the level of detail and complexity that can be considered in practical studies. In scenario-specific studies that may follow, more elaborate evaluation methods can be employed.

In the case study in this paper, our focus is on relatively short disruptions, lasting between a few hours and a few days at the most. We use delays, referring to both waiting times and increases in actual travel time, in relation to the ideal situation as an indicator for the impacts of disruptions. In the delay calculations we only consider changes in route choices and not changes in destination choices, mode choices or trip frequencies, which may or may not be feasible responses in a real situation but are challenging to model. Empirical evidence from unplanned network disruptions tells us that the most common responses by individuals are changes in departure time and route choice. To a lesser extent people cancel or consolidate (mainly non-work) trips, whereas people are relatively reluctant to change travel mode

(Wesemann et al., 1996; Cairns et al., 2002; Zhu et al., 2010). Thus, in particular given the moderate duration of the disruptions, this assumption should be relatively realistic.

To calculate the delays, we use the model from Jenelius (2009, 2010a). The model incorporates disruptions whether or not users are able or unable to reach their destinations during the closure. For area-covering disruptions this is important, since some OD relations may have available routes while others may not. The model is applicable to very large, mainly uncongested road networks, where computation time and memory consumption are important issues. In particular, it is assumed that the closure of a link does not affect the travel time on any other link. This approximation is reasonable for most of the Swedish road network used in the case study. Even in densely populated areas, the congestion effects of area-covering disruptions may be smaller than one would initially expect, since users who are unable to travel during the closure will not contribute to congestion. This issue is discussed further in Section 5.

During the disruption, there may or may not be usable routes from i to j . If there are available routes, a user can choose to travel along the new shortest route or to wait until the links in c are reopened if this means reaching the destination faster. We thus assume that the users know the difference in travel time between the new and the original shortest route, denoted Δt_{ij}^c , and the closure duration, denoted τ . If there are no alternative routes, which can be expressed as $\Delta t_{ij}^c = \infty$, the best a user can do is to wait until the closure is lifted. Assuming that the travel demand is evenly or randomly distributed over time, a user wishing to depart during the closure will on average be delayed $\tau / 2$ time units. It can then be shown (see Jenelius 2010) that the total delay ΔT_{ij}^c during the closure is

$$\Delta T_{ij}^c(\tau) = \begin{cases} x_{ij} \Delta t_{ij}^c \left(\tau - \frac{\Delta t_{ij}^c}{2} \right) & \text{if } \Delta t_{ij}^c < \tau, \\ \frac{x_{ij} \tau^2}{2} & \text{otherwise.} \end{cases} \quad (3)$$

The impact model makes some strong simplifying assumptions, e.g. that the users are informed of the duration of the closure and behave optimally according to that information. Jenelius (2007) develops a model that takes the slow spreading of information about a closure into account, and shows that the resulting delays are larger in absolute numbers than with immediate information, but that the relative differences between closures of different links are virtually the same with both models, given the same closure duration in both cases. Thus, the assumption of a known closure duration should give a conservative estimate of the delays, but should not create any considerable bias between cells.

2.4 Decomposition of impacts

The importance and exposure measures can be decomposed based on whether the trip origins and destinations are located within or outside the disrupted cell, and it is interesting to consider the contributions of the different impact components in the analysis. Here we focus on the importance measure, but the decomposition of the exposure measure is analogous. Let $i \in c$ mean that origin/destination i is located within cell c . Then

$$I(c | \tau) = \underbrace{\sum_{i \in c} \sum_{j \in c} \Delta C_{ij}^c(\tau)}_{\equiv I_{\text{inter}}(c | \tau)} + \underbrace{\sum_{i \in c} \sum_{j \notin c} \Delta C_{ij}^c(\tau)}_{\equiv I_{\text{out}}(c | \tau)} + \underbrace{\sum_{i \notin c} \sum_{j \in c} \Delta C_{ij}^c(\tau)}_{\equiv I_{\text{in}}(c | \tau)} + \underbrace{\sum_{i \notin c} \sum_{j \notin c} \Delta C_{ij}^c(\tau)}_{\equiv I_{\text{through}}(c | \tau)}. \quad (4)$$

Here, $I_{\text{inter}}(c | \tau)$, $I_{\text{out}}(c | \tau)$ and $I_{\text{in}}(c | \tau)$ are the components of the total impact due to *internal*, *outbound* and *inbound* travel demand of the cell, respectively. Following Jenelius et al. (2006), we refer to the users who are unable to travel during the closure (excluding any who voluntarily postpone their trips) as *unsatisfied demand*. Since all links intersecting the cell are closed, none of these users will be able to travel during the disruption. Hence, these components are formed entirely of unsatisfied demand. $I_{\text{through}}(c | \tau)$, finally, is the component due to *through-going* travel demand, normally passing through the cell. If congestion effects would be considered, this term would also include any other trips indirectly impacted through increased or, actually, decreased congestion.

Some of the through-going demand may have alternative routes during the disruption while others may not. Hence, $I_{\text{through}}(c | \tau)$ can be further decomposed as

$$I_{\text{through}}(c | \tau) = \underbrace{\sum_{i \notin c} \sum_{j \in c | \Delta t_{ij}^c = \infty} \Delta C_{ij}^c(\tau)}_{\equiv I_{\text{unthr}}(c | \tau)} + \underbrace{\sum_{i \notin c} \sum_{j \in c | \Delta t_{ij}^c < \infty} \Delta C_{ij}^c(\tau)}_{\equiv I_{\text{detr}}(c | \tau)}, \quad (5)$$

where $I_{\text{unthr}}(c | \tau)$ is the component of the impact due to *unsatisfied through-going* travel demand and $I_{\text{detr}}(c | \tau)$ is the component due to *detours* and other adjustments.

With the impact model in (3) we get

$$\begin{aligned} I(c | \tau) &= I_{\text{inter}}(c | \tau) + I_{\text{out}}(c | \tau) + I_{\text{in}}(c | \tau) + I_{\text{unthr}}(c | \tau) + I_{\text{detr}}(c | \tau) \\ &= \frac{\tau^2}{2} \sum_{i \in c} \sum_{j \in c} x_{ij} + \frac{\tau^2}{2} \sum_{i \in c} \sum_{j \notin c} x_{ij} + \frac{\tau^2}{2} \sum_{i \notin c} \sum_{j \in c} x_{ij} + \frac{\tau^2}{2} \sum_{i \notin c} \sum_{j \in c | \Delta t_{ij}^c = \infty} x_{ij} + \sum_{i \notin c} \sum_{j \in c | \Delta t_{ij}^c < \infty} \Delta T_{ij}^c(\tau). \end{aligned} \quad (6)$$

In total, the component of the impact due to unsatisfied demand is

$$I_{\text{unsat}}(c | \tau) \equiv I_{\text{inter}}(c | \tau) + I_{\text{out}}(c | \tau) + I_{\text{in}}(c | \tau) + I_{\text{unthr}}(c | \tau) = \frac{\tau^2}{2} \sum_i \sum_{j | \Delta t_{ij}^c = \infty} x_{ij}. \quad (7)$$

Hence, with the impact model (3), all components involving only unsatisfied demand increase proportionally to the square of the closure duration.

3. Case study

3.1 Specifications and data

We have performed a grid-based vulnerability analysis of the Swedish road transport system, where the national borders define the study area. In this study we have used square cells to represent the disrupting events. There were two main reasons for using square, rather than

hexagonal, cells. First, square grids are considerably easier to create and work with using GIS raster techniques. Second, we hope in the future to estimate relative probabilities of disruptive events in different cells. It is likely that such data would be available in (square shaped) raster form.

In general, the duration of the closure is assumed to be 12 hours. To test the influence of the closure duration on the results, we have also studied a 48-hour disruption. To represent events of varying spatial extent, we have used grids of three different cell sizes: $12.5 \times 12.5 \text{ km}^2$, $25 \times 25 \text{ km}^2$ and $50 \times 50 \text{ km}^2$, respectively. These cell sizes, where even the smallest can be considered relatively large considering the closure durations, were chosen more in order to study the impact of the size of the disrupted area than to represent any specific type of hazard. Still, such disruptions may be caused by for example heavy snowfall or storm felled trees, where the links can be cleared and reopened fairly rapidly.

For the two smaller cell sizes we have used four grids each, symmetrically displaced in two longitudinal and two latitudinal steps as illustrated in Figure 1. For the 50 km cell size we have used 16 grids, evenly displaced in four longitudinal and four latitudinal steps. While each grid was originally created with a fixed number of columns and rows and slightly larger than the study area, we only consider cells that intersect the study area in the analysis. The mean number of cells per grid intersecting the study area is 3170 for the 12.5 km cells, 853 for the 25 km cells and 241 for the 50 km cells. To illustrate the size of the events that we consider in relation to the study area, Figure 2 shows a portion of one of the 25 km cell grids.

Figure 2 about here.

The network and travel demand data have been obtained from the Swedish national travel demand model system SAMPERS (Beser and Algers, 2001). The SAMPERS system divides Sweden into zones in which all trips begin and end, each zone comprising about 1,000 inhabitants. Travel demand between different zones is calculated using nested logit choice models which have been estimated on travel surveys. Trips are loaded onto the network through centroid nodes that serve as origins/destinations, attached to the network with connector links. The travel time of each link in the original undisrupted network is calculated with user equilibrium traffic assignments in EMME/2 (using a 0.001 relative gap stop criterion), which means that initial congestion is considered in this study. The OD travel demand matrix used in our study represents the annual daily average travel demand and includes both car and truck trips. The demand is kept fixed in the present analysis. In other words, the users are assumed not to cancel their trips as a reaction to disrupted links.

The road network representation consists of 32759 nodes (including 8764 OD nodes) and 86940 directed links, and represents a very fine level of detail.² A few links in Norway and Finland have been added to provide alternative routes and reduce border effects, but are not closed in the vulnerability analysis; neither are the ferry links to the island of Gotland in the southeast. Since we do not have data on the curvature of the links, they are represented as straight lines. This can have some impact on the results in that long links in particular may intersect the wrong cells, but the significance of this effect is small overall.

² The network originally consists of 77769 nodes and 174044 directed links. We have been able to reduce the size of the network without affecting the results of the analysis by removing dead-end nodes (not OD nodes) and replacing links connected in series with a single link. The original network is used to calculate the road density at the municipal level presented below.

The projection of the three-dimensional surface of the Earth onto a two-dimensional map and vice versa necessarily creates some distortion of shapes and areas. The maps of the study area and the road network use the RT90 coordinate system, which is based on a Gauss-Krüger transverse Mercator projection (see e.g. Snyder, 1987). This projection creates little distortion, particularly in the latitudinal direction, and we have verified that the differences in shape and size between cells in the true three-dimensional space are negligible.

The road network representation is shown to the left in Figure 2. To the right, each of the 8764 origin/destination nodes is shown with the size reflecting the total travel demand originating from the node to all destinations. The map also shows the population density at the county level. As can be seen, the road network, population and travel demand are considerably denser in the southern parts of the country. In the north, travel is mainly concentrated to the eastern parts near the coast line of the Baltic Sea.

3.2 Computations

The analysis procedure relies heavily on GIS techniques. GIS software (ArcGIS 9.2) was used to create the grids, to identify all cells intersecting the study area, to identify all links and origins/destinations intersecting each cell, and to calculate the length proportions of each link intersecting each cell. These data were imported into specially developed software written in C++/C#, where the impact calculations were performed. The results were then returned to the GIS for visualization. For the analysis method to be practically applicable in the future, we believe that all steps of the process should be integrated into the same work environment, for example by calling the impact calculations from within the GIS.

In the delay calculations, the original travel times from each origin to all destinations were calculated using Dijkstra's algorithm implemented with approximate buckets (Cherkassky et al., 1993), which was determined to be particularly fast for road networks. This algorithm generates a shortest-path tree with the origin as root. To calculate the delays of a cell disruption, the straightforward method is then to close all intersecting links and recalculate the travel times from scratch. We found, however, that it was typically considerably faster to close the intersecting links progressively and update the shortest-path tree after each additional link had been closed. This procedure was based on a reoptimization algorithm adapted from Buriol et al. (2004). Link closures were implemented by setting link travel times to a sufficiently large constant, which allowed us to identify unsatisfied demand as destination nodes with travel times from the origin larger than the constant. Using a 2.0 GHz Pentium III 1 GB laptop, calculating the the importance of every cell and the exposure of every county for a specific grid took about 2.5 hours for the 50 km cell grids up to about 10 hours for the 12.5 km cell grids.

4. Results

4.1 Cell importance

Some main results from the cell importance analysis are summarized in Table 1. Not surprisingly, the mean and median cell importance increase with the cell size.³ For each cell

³ If we include the theoretical data point (0, 0), meaning that mean cell importance tends to zero as cell size tends to zero, the mean cell importance follows very closely a second-order polynomial in cell side length with the non-zero root being negative. The explanation for this is that the quadratic term represents internal travel

size, a few cells are of extremely high importance in relation to the vast majority, which means that the distributions of importance among cells have very long tails. An effect of this is that the median cell importance is very much lower than the mean. The coefficient of variation (i.e. the standard deviation divided by the mean) between cells decreases with the cell size. This is reasonable since more of the spatial variability in the transport system should be contained within cells rather than appear between cells.

Table 1 about here.

Table 1 also shows the different components of the total impact (see Section 2.4). The mean share due to internal travel demand (i.e. within the disrupted cell) increases, while the mean shares due to outbound, inbound and unsatisfied through-going travel demand appear to stay relatively stable. The mean share due to detours and/or voluntary postponements decreases from about 40% for the 12.5 km cells to about 10% for the 50 km cells; in other words, the mean share due to unsatisfied demand (i.e. involuntarily postponed trips) increases from 60% to 90%.

For the 25 km cell grids, Figure 3 shows how the shares of the different impact components vary when the cells are sorted by increasing importance from left to right. The values on the x-axis represent the relative importance ranking of the cells, that is, the percentage of all cells that are of less or equal importance. To make the diagram clearer, the shares are aggregated to mean values over each 5% interval; hence, the x-axis starts at 5%, representing the 5% least important cells.

Figure 3 about here.

As can be seen, for the 75% most important cells (i.e. at the 25% point on the x-axis) the travel demand of the cell itself (i.e. internal, outbound and inbound travel) is typically responsible for the major part of the impact. If in addition we consider the through-going unsatisfied travel demand, it is clear that almost the entire impact is caused by unsatisfied demand, in particular for the 50% most important cells. The corresponding plots for the 12.5 km cell grids and the 50 km cell grids differ mainly in that the curves are shifted about 10 percent units to the right in the former case and about 20 percent units to the left in the latter case.

Since the impact per unsatisfied trip is the same for all cells for a given closure duration (compare (5)), this finding implies that the duration has relatively little impact on the relative importance of different cells. The qualitative results of studies of this kind should therefore be insensitive to what closure duration is assumed, although the impacts in absolute numbers of course are not. We have confirmed this conclusion by comparing the results for the 12-hour duration with those for a 48-hour duration.

To illustrate the skewness of the importance distribution among cells, Figure 3 also shows the cumulative importance distribution of the 25 km cells. Thus, for a given percentage p on the x-axis, the curve shows the sum of the importance for the p least important cells. In economic equality analysis, this is known as the Lorenz curve. The curve reveals that, for example, the 5% most important cells are responsible for about 50% of the total impacts across all cells.

demand, which should increase proportionally to the area of the cell on average (and hence quadratically with the side length). In addition, inbound and outbound travel demand should increase proportionally to the circumference of the cell on average, which gives rise to a positive linear term and hence the negative root.

Figure 4 shows the geographical distribution of cell importance for the 12.5 km cell grids to the left and the 25 km cell grids to the right. For each point in the study area, each map shows the mean importance of the four cells covering the point. This approach allows us to present the results across all four grids on the same map, which has a resolution of half the cell size, i.e., 6.25 km squares for the 12.5 km cells and 12.5 km squares for the 25 km cells (compare Figure 1).

Figure 4 about here.

By comparing the results with Figure 2, we see that the geographical distributions of cell importance show high correlation with the concentration of travel demand. The main cities in northern Sweden are clearly visible, as are the urban areas in the south. In general, cell importance is concentrated to the southern regions; note also the extreme variation in importance between cells as indicated in the map legends. Details of the road network are more apparent in the results for the smaller 12.5 km cell grids (compare Figure 2). For the larger cells, the properties of the road network seem to be of less significance than the location patterns for the magnitude of the impact. Indeed, for the larger 50 km cell grids the features are even more smoothed out and the distribution resembles the general variations in population density.

The increasing correlation of cell importance with location patterns as the cell size increases (and the decreasing correlation with network density) can be understood from the observation above that the impacts become increasingly related to the internal, outbound and inbound travel demand of the cell. Since the travel demand is highest at locations with a big and dense population, it follows that the impacts will be the most severe at these locations. In contrast, a closure of the links in an area with low population concentration will mainly lead to detours for through-going travel demand, which results in less severe impacts. For smaller cell sizes fewer links will be contained in each cell, and the vulnerability will increasingly resemble that for single link failures as the cell size decreases.

4.2 Area-covering disruptions vs. single link failures

To better understand the characteristics of area-covering disruptions, we have studied these in relation to the impacts of single link failures. For each cell, we have compared the impact of closing all intersecting links (i.e. the importance of the cell) with the average impact of closing a road segment of unit length within the cell. This latter measure corresponds to the “regional importance” measure introduced by Jenelius (2009), where the “region” in this case is the cell. The measure is equivalent to the expected impact of closing a randomly selected link intersecting the cell, with selection probabilities proportional to the lengths of the parts of the links that are inside the cell.

The ratio of the two importance measures is shown to the left in Figure 5 for all cells in the 25 km cell grids as a local mean of the ratios across the four grids. As the legend shows, the impacts may differ by several thousand times. The lower bound of 1 means that closing all links has the same effect as closing a randomly chosen road segment, which occurs when there is only one or, sometimes, a few links intersecting the cell. As can be seen, the ratio between the impacts of area-covering disruptions and single link failures is particularly large in the southernmost region. Many of the “hot spots” at population centres appearing in Figure 4 are not as visible in Figure 5, particularly in the north. This suggests that the high

importance of these cells is caused by one or a few key links being disrupted, a fact that we have confirmed by more detailed study.

Figure 5 about here.

To the right in Figure 5, the density of the road network is shown at the municipal level, measured as the length of the road network in an area divided by the size of the area. Comparison of the two maps shows that there is a strong spatial correlation between high cell/mean link importance ratios and a dense road network. This can be understood from the fact that road density, in turn, is strongly correlated with population density (see Jenelius, 2009). Jenelius (2009) showed that while the high link flows in regions with dense road networks typically lead to many users being affected by single link failures, the good availability of alternative routes means that the average delay per user is small.

Area-covering disruptions, on the other hand, will have severe impacts for at least two reasons. First, disruptions may affect many links that would act as parts of alternative routes for each other given only a single link failure, leading to longer detours around the whole area (compare with Jenelius (2010b), who studies the importance of links as rerouting alternatives under single link failures). Second and more importantly, a high population density typically means that there is a high demand for travelling within, into, out from or through the affected area. As we argued above concerning the cell importance, the disruption means that all this travel demand will be unsatisfied, which leads to large overall delays. Hence, the differences between single link failures and area-covering disruptions will be the most drastic in regions with both a dense population and a dense network.

4.3 Worst-case regional user exposure

We have studied the worst-case user exposure of each county in Sweden for the 12.5 km cell grids. The analysis shows that the exposure is determined by how concentrated traffic flows are towards a central “hub” in the region. If a large share of the trips begin or end within a limited area, then a disruptive event centred on that area will have severe consequences for the regional traffic. If the traffic is more distributed around the region, a single disruption will affect a smaller share of travellers. This can be illustrated by comparing the location and travel patterns of one of the least exposed counties and one of the most exposed counties, as shown in Figure 6.

Figure 6 about here.

To the left is shown the least exposed county (Dalarnas län) with a worst-case average delay of 1.45 hours per traveller, and to the right is shown the third most exposed county (Jämtlands län, located just north of Dalarnas län) with a worst-case average delay of 3.50 hours per traveller. This county is chosen for the comparison since it is larger than the least exposed county, which eliminates the effect that smaller regions tend to be more user exposed (see further Section 5.2).

It can be seen that the low exposure region to the left contains a cluster of population centres in the southeast corner, which is enough dispersed in space so that a single cell can only disrupt a relatively small portion of the regional travel demand. Thus, in the worst-case scenario, 24% of the travel demand is unsatisfied. In the high exposure region to the right, meanwhile, there is a much more dominant hub in the centre of the region (the city of Östersund). Although it may not be immediately apparent from the figure, the worst-case

scenario, centred on this hub, causes more than 58% of the regional travel demand to be unsatisfied. In both regions, the impacts are about evenly distributed between internal travel demand and inbound or outbound travel demand of the worst-case cell.

5. Discussion and conclusion

5.1 General conclusions

We have presented an approach to systematically analyzing the vulnerability of road networks under disruptions covering extended areas. The methodology involves covering the study area with grids of uniformly shaped and sized cells, where each cell represents the extent of an event disrupting any intersecting links. Since various kinds of events including floods, heavy snowfall, storms and wildfires can cause such wide-spread degradations, the analysis method is an important complement to the existing studies of single link failures.

The application to the Swedish network shows that the factors determining where area-covering disruptions will have severe consequences are quite different from those of single link failures. As a result, the vulnerability of the road network against each of the two types of degradations shows markedly different geographical distributions. In the latter case, the flow on the link and the availability of alternative routes, i.e. the local redundancy in the network, largely determine the magnitude of the impact. For disruptions covering extensive areas, the network redundancy has a much smaller influence, since nearby potential alternative links are often also disabled. It is almost entirely the unsatisfied travel demand within, into and out from the affected area itself that determines the impact. Travel demand, in turn, is closely correlated with population size, which makes regions with the population concentrated to a few central locations particularly exposed to this kind of event.

These conclusions should be relevant for not just the particular study area but for any area with similar location patterns and road network structure. Generally, the more dispersed the population and the sparser the road network in the study area, the more the impacts will resemble those of single link failures. The more congested the road network, on the other hand, the more the differences between single link failures and area-covering disruptions will increase also in areas with mainly through-going traffic. Conclusions may be qualitatively different if one would consider, for example, more confined disruptions in a congested city network, or longer durations of the hazards. Applying the proposed grid-based analysis methodology in such settings, combined with appropriate impact models, are interesting directions for future research. The methodology could also be adapted and applied to other transport systems, including the railway and the aviation systems, and for multi-modal systems.

5.2 Model limitations

A full range vulnerability analysis approach, such as the present one, is suitable for gaining an overview of the situation in the study area, making comparisons between different regions and drawing general conclusions about the factors underlying vulnerability. We are therefore interested in the relative impacts of different cells, whereas the impacts in absolute numbers are less critical to assess precisely. We have attempted to make simplifying assumptions in the model in a way that introduces as little systematic bias between different parts of the study area as possible. We use a stylized model of the disruptive event, in which there is a precise

spatial and temporal limit for the disruption. We also assume that users have complete information about the network and the disruption and behave optimally under this information (note that the general grid-based approach is not dependent on this assumption). Although any real event and people's responses to it would be much more complex, we do not believe that the assumptions distort the general picture in any fundamental way, as long as we consider events that degrade a significant portion of the road network within their extent.

The presented approach has some limitations that should be noted. Even though we can control the level of detail in the analysis, we only consider a finite number of possible locations for the event. This means that there may be other locations where an event would have more severe impacts, since a somewhat different set of links would be affected. Also, an event of the same size but slightly different shape may have different impacts. This is especially important to remember in worst-case analyses, in which the identified worst-case scenario by construction is confined to the predefined set of considered events.

The results of regional exposure analyses are also quite sensitive to how the regions are defined. First, there is a bias towards small regions being more exposed than large regions. This is because a disruptive event covers a larger fraction of a small region than a large, and potentially a larger share of the generated travel demand. Second, depending on how the regional borders are drawn, a certain population centre may fall just within or just outside a particular region, which could have a large influence on the impact for the region (this is an instance of the common problem in geographical studies known as the "modifiable areal unit problem"). To reduce these problems we performed the exposure study at the county level rather than e.g. at the finer municipal level, where the size bias is evident in the results. Furthermore, the county borders have a clear economic and political role, which makes comparisons between different these regions relevant.

We assumed in the case study that travel times on links outside the disrupted area are not affected by the change in traffic flows that the event causes (again, the grid-based approach is not dependent on this assumption). In general, the number of users who are able to take detours during the disruption is much smaller than the number of users who are unable to travel. It seems plausible that the net effect for other users would be a reduction in travel time rather than an increase, since users who are unable to travel will not contribute to congestion. On the other hand, there may be considerable congestion when the closure is lifted and all users are able to travel again. It is difficult to say whether these effects together would have any considerable influence on the relative impacts of different cells.

5.3 Policy implications

An important reason for doing vulnerability analyses is the possibility to take preventive actions to reduce the frequency with which disruptions in the network occur or to reduce the negative impacts if disruptions occur. The fact that the regions that are most vulnerable to area-covering disruptions in general are not the same as those that are most vulnerable to single link failures suggests that each type of hazard must be evaluated separately and then be synthesised in the later stages of the assessment.

The possibility to affect the frequency of disruptions depends much on the kind of events that is considered. For example, events like flooding could in some cases be avoided by building dams and fortifications. Many other area-covering events, e.g. heavy snowfall and storms, may be difficult or impossible to prevent. The focus in the planning must then primarily be on

minimizing the negative effects on the operability of the road network and of the consequences of the reduction in operability that cannot be avoided. Given the limited influence of the network structure on the impacts, we believe that reducing vulnerability to this kind of events is more an issue for maintenance and operations than for redundancy-providing infrastructure investments.

Our case study shows that unsatisfied demand, i.e. trips that cannot be made, constitutes nearly all of the impact on travel during a serious disruption. This impact increases rapidly (quadratically in our model) with the duration of the disruption. Hence, an efficient way to reduce the vulnerability would be to restore the disrupted links as quickly as possible. The study also shows that the impact increases rapidly with the size of the disrupted cell. This implies that even if a disruption cannot be avoided completely, it is valuable to limit its geographical extent as much as possible.

As for any rare events, estimating the frequencies with which different kinds of disruptive events can be expected to occur in different parts of the study area is a difficult task, and good estimates may depend on specific environmental features of each area. However, the likelihood of flooding in an area, for example, is certainly influenced by the precipitation, for which detailed historical or modelled future data on both average and extreme levels are often available. Moreover, data for this and other meteorological statistics are not uncommonly produced in the form of square grids, which makes the integration with our impact calculations straightforward. Such development of the methodology for likelihood assessments in vulnerability analysis should be an important area for further work.

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Figure captions

Figure 1: Illustration of four superimposed square cell grids.

Figure 2: Left: The Swedish road network representation and a portion of a 25 km cell grid. Right: Total outbound travel demand for every origin/destination node and the population density of every county.

Figure 3: Areas: Shares of the total impact due to different demand components (%) for every 25 km cell sorted by increasing importance, aggregated as mean values over every 5% importance interval. Line: Cumulative cell importance (%), i.e. the Lorenz curve. 12 h closure duration.

Figure 4: Local mean cell importance (veh. h) for the 12.5 cell grids (left) and the 25 km cell grids (right). 12 h closure duration. The percentage of the cells within each category is indicated in parentheses.

Figure 5: Left: Local mean ratio between cell importance and length-weighted average link importance for the 25 km cell grids (12 h closure duration). The percentage of the cells within each category is indicated in parentheses; cells with undefined ratios are not shown. Right: The road density (km/km^2) for each municipality in the study area. The categories represent deciles.

Figure 6: Illustration of the travel and location patterns of a low exposure region (left) and a high exposure region (right). OD relations with travel demand less than 1 veh/h are not shown.

Table captions

Table 1: Results from the cell importance analysis (12 h closure duration).

Table 1

Cell size (km)	Importance (veh. h)			Demand components of impact (%)				
	Mean	Med.	Coeff. of var.	Internal	In+out	Through unsat.	Through detours	
12.5	14600	224	4.90	4.20	49.2	7.15	39.4	
25	47500	3790	3.57	13.5	60.2	5.73	20.5	
50	148000	19700	2.51	30.6	56.1	3.77	9.56	

Figure 1

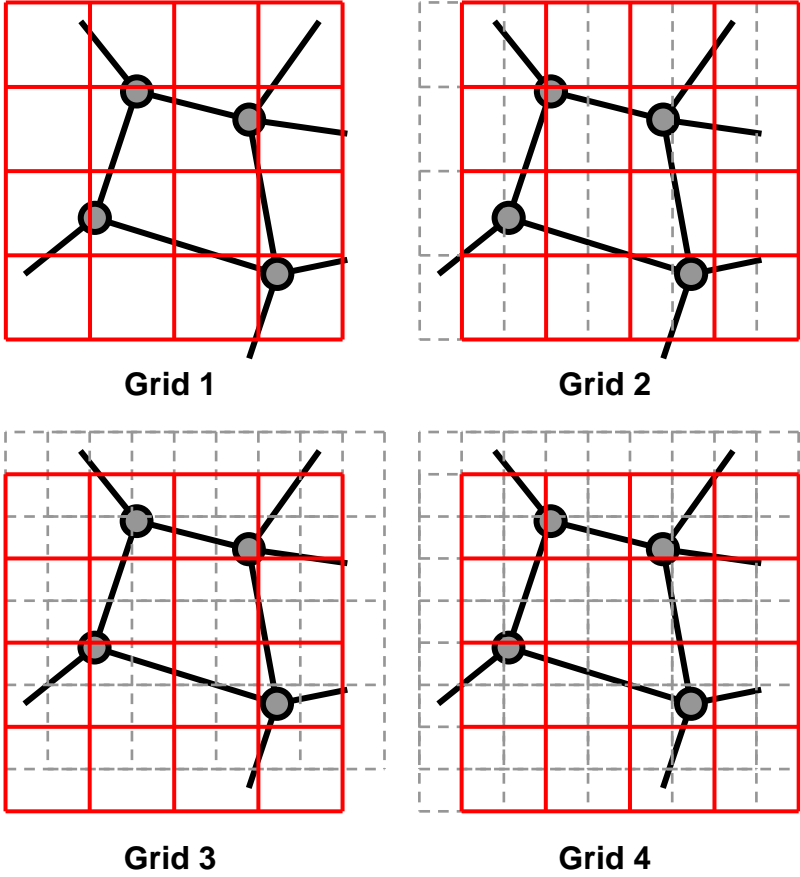


Figure 2

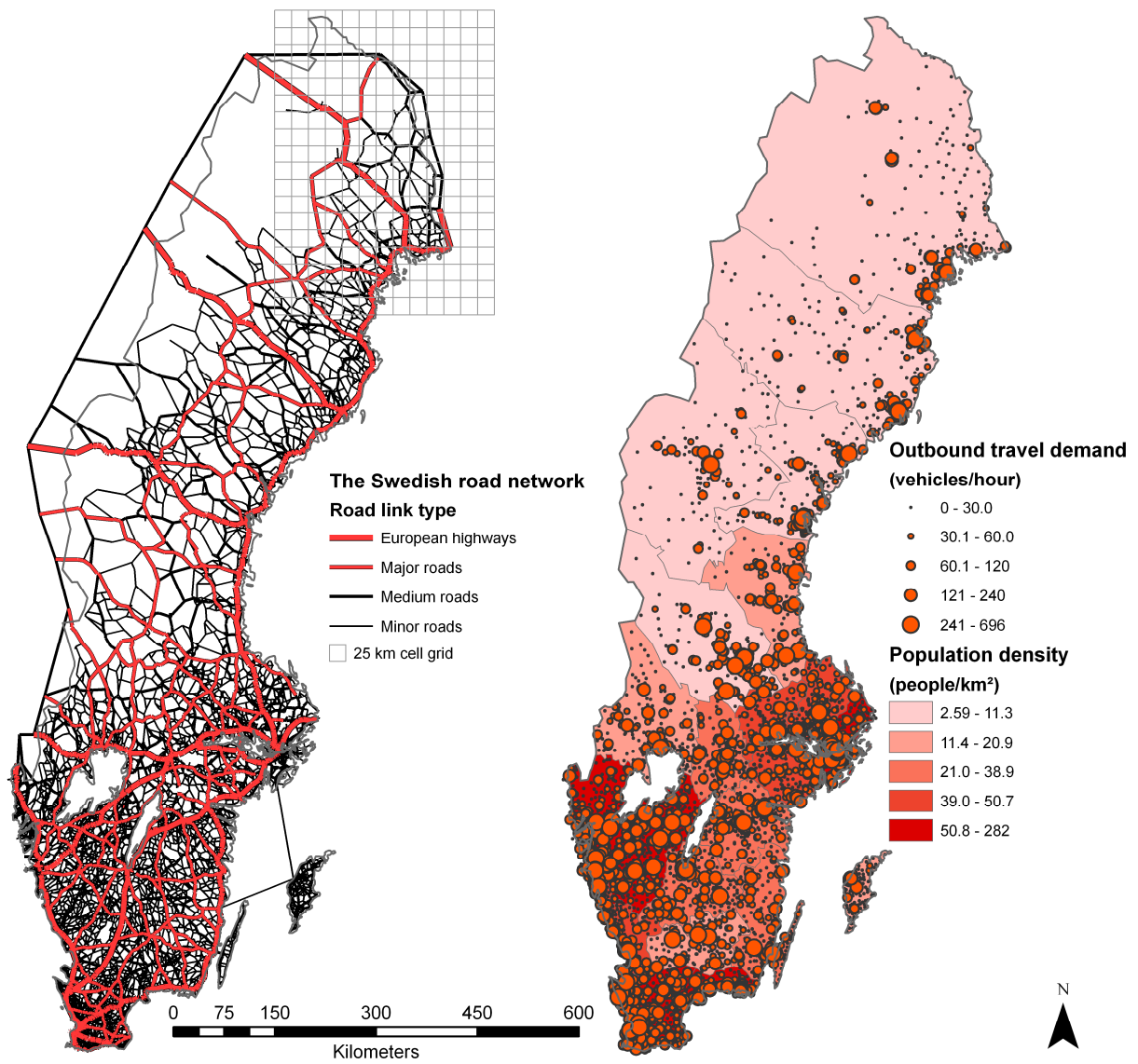


Figure 3

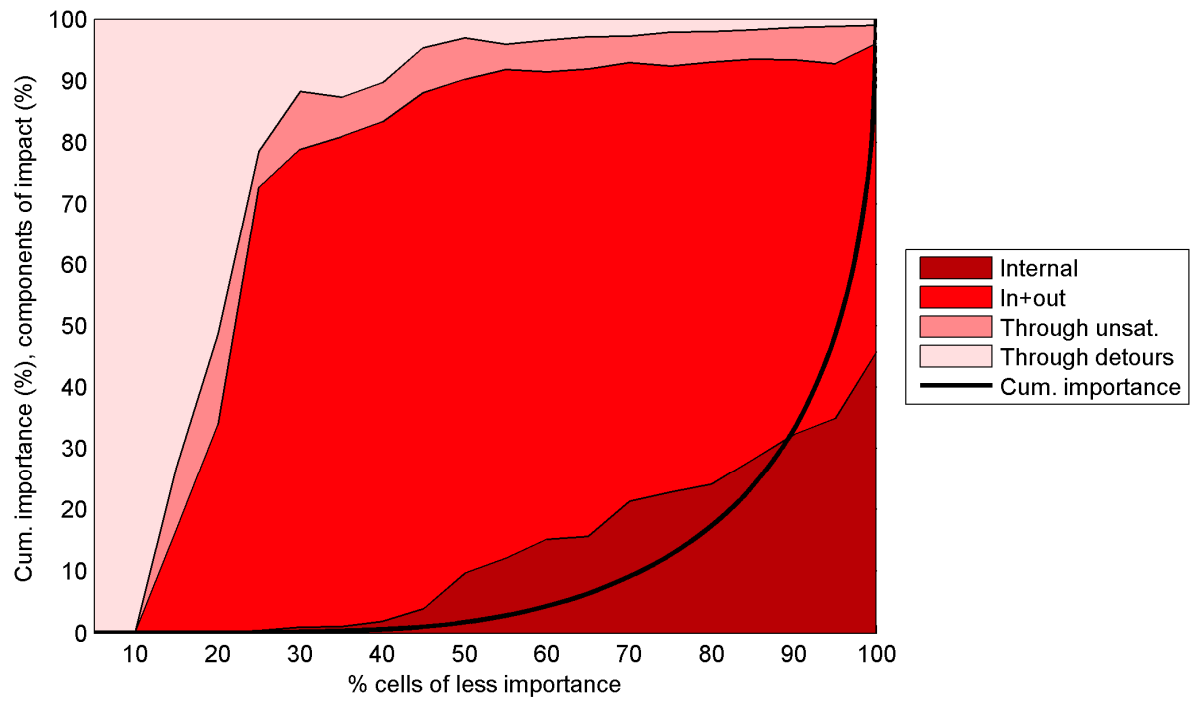


Figure 4

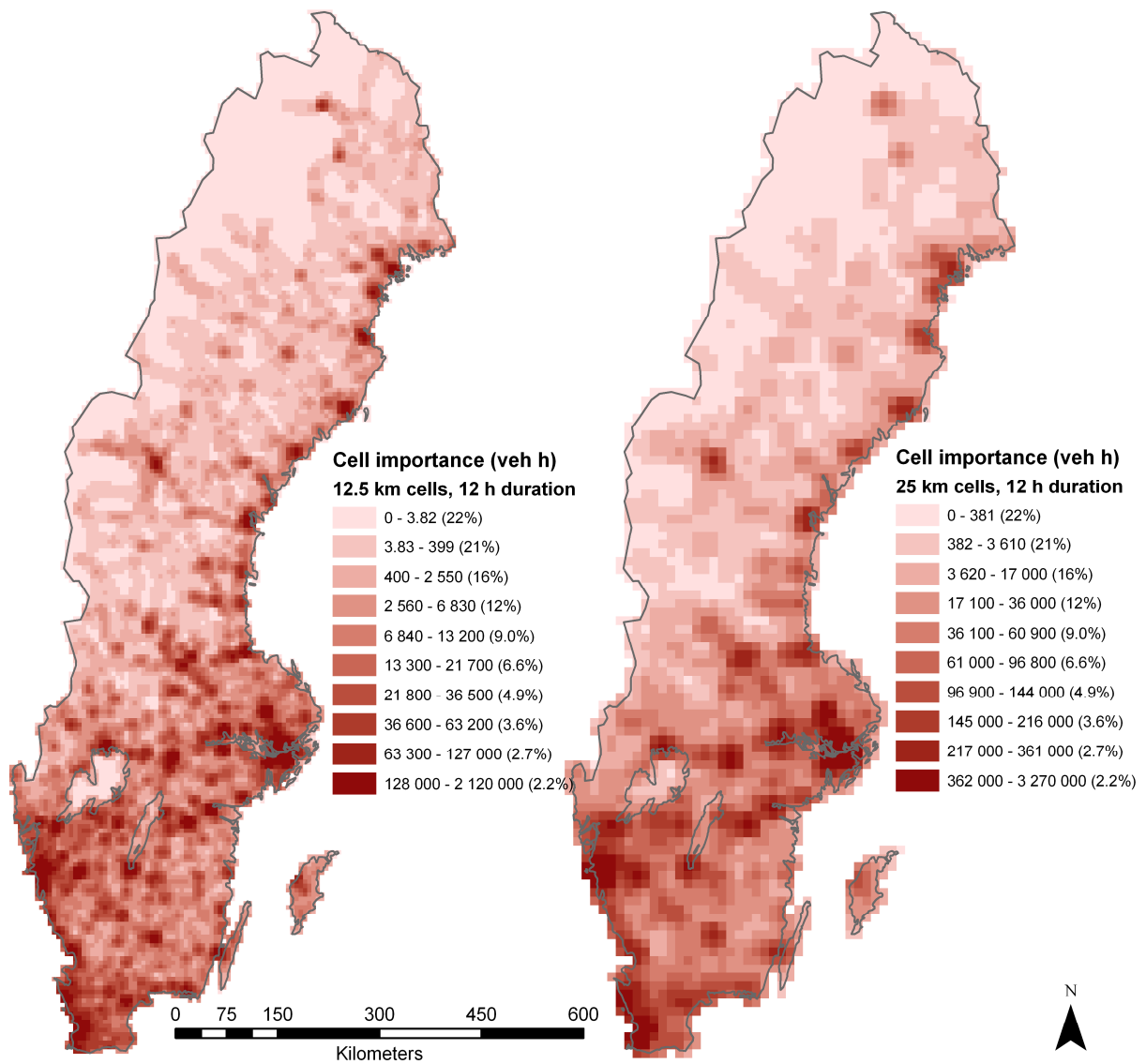


Figure 5

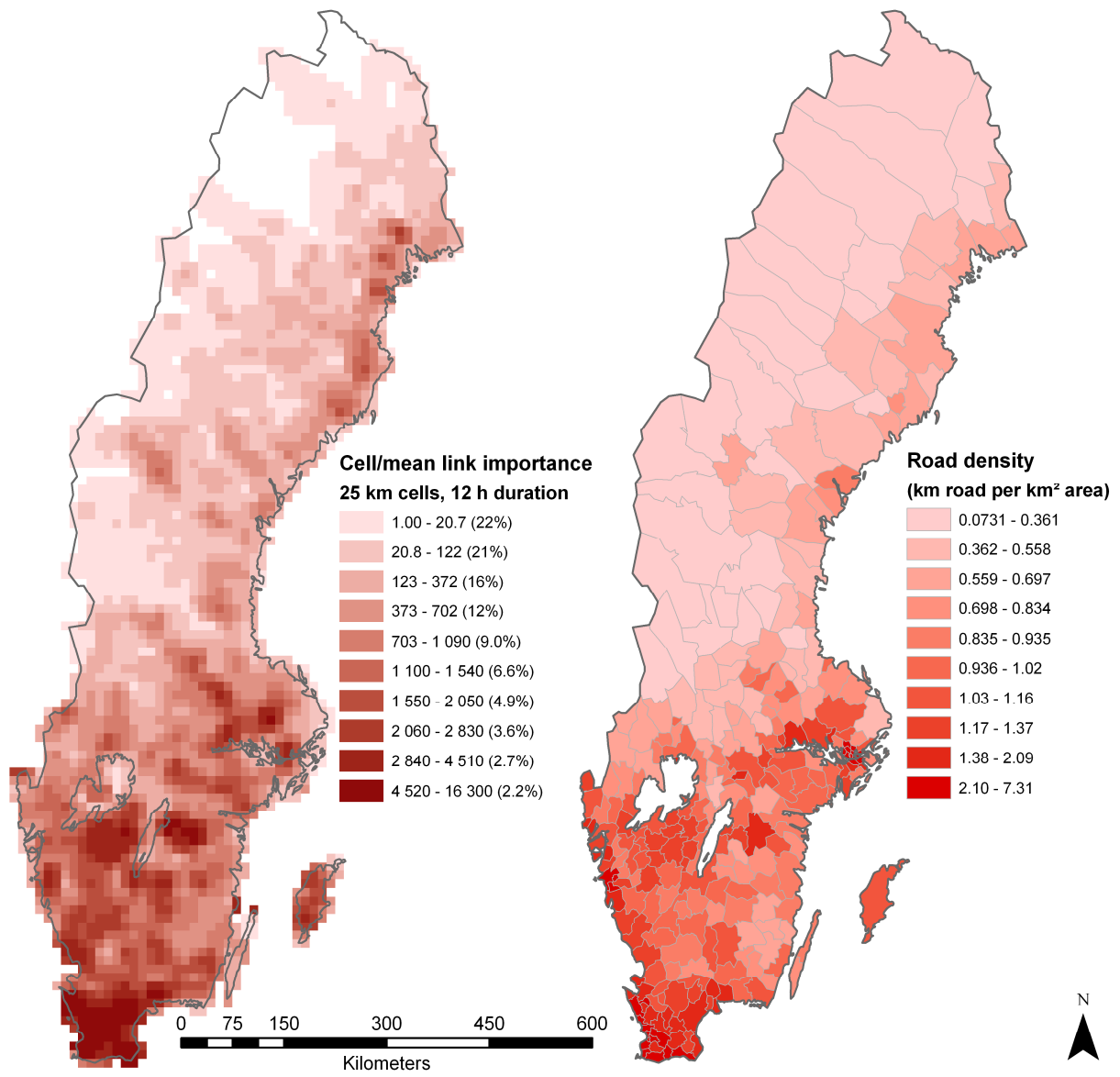


Figure 6

