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IMPORTANCE AND EXPOSURE IN ROAD NETWORK VULNERABILITY ANALYSIS: A CASE STUDY FOR NORTHERN SWEDEN

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ABSTRACT

The reliability and vulnerability of critical infrastructures have attracted a lot of attention recently. In order to assess these issues quantitatively, operational measures of the importance of links and the exposure of sites in infrastructure networks are needed. Such measures can also be used as a guidance to road administrations in their prioritisation of maintenance and repair of roads, as well as for avoiding causing unnecessary disturbance in the planning of road work. In this paper, several link importance indices and node exposure indices are derived based on the increase in generalised travel cost when links are closed. These measures are calculated for the road network of northern Sweden. Results are collected in a GIS for visualisation, and are presented per link and municipality.

KEYWORDS: vulnerability, reliability, criticality, exposure, road network.

1. **INTRODUCTION**

For at least a decade, there has been a research interest in questions of vulnerability and reliability of transport networks. Events that have spurred this interest are the earthquake in Kobe in 1995 and the terrorist acts of September 11, 2001. Given the importance of the transport networks for journeys to work, production logistics (notably the *just-in-time* production philosophy), and business travel, the reliability of transport networks is a key interest from the point of view of transport system users and hence planners at all levels, both in the public and private sectors. In incident and contingency planning and the planning of road works, there should be an awareness about the impacts of the reduced capacity on different links. Two of the strategic goals of the Swedish Road Administration (SRA) are "regional balance" and "an accessible transport system", for both citizens and businesses. A relevant question to ask is therefore: "Which regions are most susceptible to disruptions in the transport system?" This necessitates some general measures of *importance* of links in the network, and *exposure* of sites, municipalities etc.

For the purpose of road management, prioritisation for road maintenance and repair, and for the assessment of regional disparities, it would thus be helpful with a general "vulnerability index" attached to links and nodes. In the following chapter, we attempt to develop such measures. We argue there that the vulnerability concept can be divided into two parts, one containing the probability of a hazardous event and the other, which we call *exposure*, containing the consequences of the event in a certain place. Exposure is thus site-dependent. Similarly, following Nicholson and Du (1994), we call the consequences for a collection of sites of a failing link the *importance* of that link. As a measure of the consequences of failure we use the increase in generalised travel cost. This quantity is aggregated over nodes and/or links to yield measures of exposure and importance for single nodes and links, municipalities and the whole network.

The measures are then calculated for the national[1] road network of northern Sweden. First, the geographical context of this region is briefly described, and the data and methods that we use are discussed. We then present some interesting results from the case study, in which the most exposed municipalities and the most important links are identified.

2. **DEFINITIONS OF CONCEPTS**

2.1 **Vulnerability and reliability**

In recent years, the issue of road transport vulnerability has received increasing attention. While no firm definition exists yet, many authors agree that vulnerability is closely related to the more established concept of *reliability* (Berdica, 2002; D'Este and Taylor, 2003; Nicholson, 2003; Husdal, 2004).

In general terms, reliability "describes the *operability* of the network under varying strenuous conditions (i.e., *the ability to continue to function*)" (Husdal, 2004). A number of reliability measures focusing on different aspects of road transportation have been proposed and used in recent years (see, e.g., Bell and Iida, 1997; Berdica, 2002; Nicholson, 2003). However, Berdica (2002) notes that reliability measures are generally concerned only with probabilities

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[1] The network used by SRA for policy analysis on the national level.

and argues that the magnitude of the consequences of a disruption should also be taken into account. For this purpose she uses the term vulnerability, which she defines as “a susceptibility to incidents that can result in considerable reductions in road network *serviceability*”. Serviceability of a link/route/road network, in turn, “describes the possibility to use that link/route/road network during a given period”, where the formulation “possibility to use” includes the condition that the cost of travel is not too high. More generally, she proposes that vulnerability can be seen as the complement of reliability, or in the terms of Husdal (2004), “the *non-operability* of the network under certain circumstances”.

Both D’Este and Taylor (2003) and Nicholson (2003) note that much of the research on reliability so far has focused on dense, congested urban road networks, where the issues mainly concern fluctuations in the traffic and/or capacity of the links. The regional and national road networks outside the cities, however, are sparse and uncongested in relation. At this level the main objectives are to connect population centres, provide access to all parts of the region and give the non-urban community access to essential services. D’Este and Taylor (2003) argue that vulnerability is a more appropriate concept than reliability in these networks, since reliability measures may overlook potential problems in these networks. In particular, the probability of a link failure may be low and the effect on overall network performance small, but the consequences for particular sections of the community may be substantial.

2.2 **Criticality, importance and exposure**

In the view of Nicholson and Du (1994) and Berdica (2002), the concept of vulnerability includes both probability and consequence. D’Este and Taylor (2003), however, argue that vulnerability should only be concerned with consequence, and that a node is vulnerable “if loss (or substantial degradation) of a small number of links significantly diminishes the *accessibility* of the node, as measured by a standard index of accessibility”. Sarewitz et al. (2003) also point at the disadvantages of including the probability of failure in vulnerability studies. Estimating the probability of extreme events such as natural disasters and large accidents is very difficult. The probability is often predicted from historical data, which presupposes that the circumstances around the event have remained the same, or that all factors affecting them are known. If the probability is underestimated or overlooked completely, the effects could be disastrous.

To resolve this difficulty, we argue that vulnerability can be treated in the same way as the closely related *criticality* concept. The criticality of a certain link in the network involves the probability of the link failing and the consequences of that failure for the system as a whole. If the probability of an incident is high, the link is *weak*, and if the consequences are great, the link is *important*. If it is both weak and important, the link is *critical* (Nicholson and Du, 1994). Likewise, we need a dissociation of vulnerability into a probability and a consequence component. For very rare events where estimation of the probability is not feasible, such as terrorist attacks or nuclear power plant failures, the term *conditional vulnerability* is often used (namely, given that a hazardous event occurs). Similarly, it would be possible to use an expression like *conditional criticality* for the importance, defined as above, of a link. Alternatively, we suggest the term *exposure* as the counterpart to importance for the phenomenon of “being dependent on links of great importance”. We get a division of the concepts as shown below.

Composite concept, A∗B	**Concept A**	**Concept B**
criticality	weakness	importance *or* conditional criticality
vulnerability	probability	exposure *or* conditional vulnerability

2.2 **Measuring importance and exposure**

In the formulations above, vulnerability is defined in terms of reduced "operability", "serviceability" and "accessibility", respectively. These concepts all concern the function of the network, but accessibility approaches the issue from the demand side, whereas operability/serviceability focus on the supply side. Although these are all complex concepts, we argue that a reasonable measure of the reduced operability/serviceability/accessibility is the increase in generalised cost of travel (time, distance, money, etc.) for the users of the network. The cost of travel is surely a strong indicator of the function of and the possibility to use the network, and it is one of the standard indices of accessibility given in D'Este and Taylor (2003). Our approach is similar to the one outlined in Taylor and D'Este (2004).

We assume that the "hazardous event" affecting the network is a link being completely disrupted or closed, which forces all travellers on that link to take other, less advantageous routes[2]. The travellers are assumed to behave according to the user equilibrium principle, i.e., to choose a route from their origin to their destination that minimizes their travel cost. At the most basic level we consider an origin node *i*, a destination node *j* and a link *k*. In the following, the set of origin/destination nodes are called *demand nodes*. We denote the cost of travel from demand node *i* to demand node *j* when link *k* has failed by $c_{ij}^{(k)}$ and let $c_{ij}^{(0)}$ represent the cost of the initial, undamaged network. The quantity

$$C(i,j,k) = c_{ij}^{(k)} - c_{ij}^{(0)}$$

is the basis from which all measures of importance and exposure are derived by aggregating over the appropriate sets of nodes.

The importance of a link *k* can be calculated with respect to a single demand node, a group of demand nodes (such as those in a municipality) or the whole network. In the latter case we aggregate $C(i,j,k)$ over all origin-destination (OD) pairs. Each OD pair can be assigned a weight w_{ij} that reflects its significance in relation to the other pairs. The importance of link *k* for the whole network **N** is then

$$I_{\mathbf{N}}(k) = \frac{\sum_i \sum_{j \neq i} w_{ij} (c_{ij}^{(k)} - c_{ij}^{(0)})}{\sum_i \sum_{j \neq i} w_{ij}}.$$

If the weights w_{ij} are all equal, this becomes

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[2] This can easily be generalised to closures of groups of links and/or nodes, such as a road in its entirety.

$$\frac{1}{N^{\mathrm{d}}(N^{\mathrm{d}}-1)}\sum_{i}\sum_{j\neq i}(c_{ij}^{(k)}-c_{ij}^{(0)}),$$

where N^{d} is the number of demand nodes in the network. This is a global perspective in which all origins and destinations are equally important, representing the role of the road system to provide access to all parts of the region. If the travel demand x_{ij} is used as weight, the severity of an increase in travel cost between two nodes depends on the traffic between them. This measures the capability of the road system to provide economically efficient transports where the demand is the highest. Since in our case most trips are to nearby destinations, it also becomes a local perspective.

Similarly, exposure to a certain event can be calculated for a single demand node, a group of demand nodes or the whole network. For a municipality *m*, $C(i,j,k)$ is aggregated over all origins *i* in the municipality and all destinations *j* in the entire network. One simple event is the failure of the most important link for the municipality. The exposure of municipality *m* is then the maximum value over all links *k*,

$$E_{\max}(m)=\max_{k\in E^{\mathrm{nc}}}\frac{\sum_{i\in V_m^{\mathrm{d}}}\sum_{j\neq i}w_{ij}(c_{ij}^{(k)}-c_{ij}^{(0)})}{\sum_{i\in V_m^{\mathrm{d}}}\sum_{j\neq i}w_{ij}},$$

where E^{nc} is the set of *non-cut links*, i.e., links that do not divide the network into isolated parts when closed, and V_m^{d} is the set of demand nodes located in municipality *m*. This “worst-case” scenario would correspond to an attack by an informed antagonist who wishes to cause as much damage as possible to the municipality with only one strike.

Beside increase in travel cost, there is another, more severe, consequence that must be taken into account. When a link *k* is closed, the network may be divided into two disconnected parts. While the travel cost between nodes in the same part is unaffected by the closure, the travel cost between nodes in different parts becomes infinite. Although serious, we do not think that inability to travel is infinitely worse than any finite increase in travel cost, and we would want some finite measure of the consequences of this event. For this purpose we introduce the concept of *unsatisfied demand* $u_{ij}^{(k)}$, defined as

$$u_{ij}^{(k)}=\begin{cases}x_{ij} & \text{if } c_{ij}^{(k)}=\infty,\\ 0 & \text{if } c_{ij}^{(k)}<\infty.\end{cases}$$

It is the number of trips from origin node *i* that are unable to reach their destination node *j* due to the closed link *k*. At aggregated levels, we generally measure the unsatisfied demand relative to the total demand. How to value unsatisfied demand in relation to increased travel cost is still an open question, however.

In these measures, the demand x_{ij} between all origins and destinations is assumed to be fixed, i.e., unaffected by changes in travel cost, for the duration of the link closure. Inelastic demand is a reasonable assumption if the trip, as for the major part of our case, is a trip to work, which normally has to be pursued no matter what. With elastic demand, travellers would alter their travel decision, destination choice and/or mode choice when facing increases in travel cost or other disturbances in the network. For an accurate assessment of the economic consequences of link failures, it is essential to use elastic demand in the calculations. This is done in Nicholson (2003) and Berdica and Eliasson (2004), where accessibility in the form of user benefits is represented by the “rule-of-a-half” (see, e.g., Neuburger, 1971, p. 56 ff.).

3. **THE ROAD NETWORK OF NORTHERN SWEDEN**

3.1 **Geographic context**

Northern Sweden is generally more sparsely populated than the southern part, which historically has to do with lack of arable land and cold climate, and geologically with the Fennoscandian mountain chain along the border to Norway in the northwest. The region also borders on Finland in the northeast and the Botnian Sea and Gulf along the east coast. In the northernmost and western parts, much of the land is marsh and mountains, unusable for cultivation. These are traditionally the grounds of the native, reindeer-breeding Sami people, who also populate the tundras of Norway, Finland and Russia. However, the areas along the coast of the Botnian Sea as well as the counties of Jämtland, Dalarna and Gävleborg have become more populated due to both the climate and the access to arable land. The population density of the municipalities in the region is shown in Figure 1.

The attraction of the northernmost parts of the region has historically mostly been economic: valuable ores (iron, copper, gold and silver) have been found there, as well as fur and pine forests, and the great rivers have been used for the construction of dams for power generation and for transportation of timber to the sawmills on the coast. Nowadays the timber is transported by road and the ore by freight trains, but the importance of the waterways for communication is still visible in that many roads and railways follow the river valleys, and that the inland settlements are often located on the shores of rivers, large lakes and lake systems. There are two main roads going in the north-south direction: the E4 European highway following the coast and the inland national road 45. Going along the rivers in the northwest-southeast direction are the European highways E10, E12 and E14 and a few national roads, starting in Norway and ending on the Botnian Sea coast.

The combination of heavy road transports and the climate, which in spring causes great difficulties while the ground below the road is thawing, makes the question of vulnerability relevant also from an economic production perspective. The thawing process starts from above, and converts the ground into something like a quagmire, threatening the road surface to break up. It is not possible neither technically nor economically to construct a road that withstands thaw. During a period in spring that can last several months, the bearing capacity of the road is severely reduced, and restrictions have to be put on the volume and time of day for road use.

3.2 **Network and travel data**

The Swedish national travel demand model system Sampers (Beser and Algers, 2001) is used by authorities to forecast the effects of different transportation policies, economic development and infrastructure investments. The Sampers model system includes a representation of the Swedish road network. For local and regional trips, Sweden is divided into 8,500 zones where all trips begin and end. The centre of gravity of each zone is represented by a *centroid* node that is attached to the actual road network with a pair of *connector* links.

The Sampers model is divided into five regional submodels. In this study we use the road network of the submodel Palt, which focuses on the six northernmost counties of Sweden: Dalarna, Gävleborg, Västernorrland, Jämtland, Västerbotten and Norrbotten. This area also represents the middle and northern planning regions of SRA. In this part the network is represented at its most detailed level, reflecting the situation in 2001, with 19,392 nodes (including 1,208 centroids) and 42,956 directed links. In the rest of the country the network is represented by the national road network, which is coarser with 7,597 nodes (including 191 centroids) and 17,796 directed links. The network data contains the length, volume-delay function (the relation between traffic load and travel time), type (from European highway to tertiary county road), number of lanes and county of each link. The data also contains the coordinates of each node. By importing the data into a GIS (Geographical Information System) and combining it with other geospatial information, we have obtained the municipality and parish in which each link and node is located.

The travel demand is measured in number of vehicles on an annual daily average. The vehicles include personal cars for private and company use, personal cars in commercial traffic, and heavy trucks with and without trailer. The users only include Swedish travellers within the confines of Sweden, not trips abroad or transit trips. We have also available the results from an assignment of traffic on the road network performed by SRA using Sampers. The model includes the effects of congestion and is based on a linear combination of travel time and distance. More precisely, the travel time is valued at 136 SEK per hour and the distance at 1.30 SEK per kilometre. Figure 2 shows the traffic load on each link in the network according to the assignment. The network here is a simplified version of the original network that will be introduced in the next chapter.

4. MEASURES AND METHODS

In the following case study, the travel times t_{ij} between the demand nodes are used as the generalised cost of travel. A perhaps more realistic model would be to use a linear combination of travel time and distance. The drawback of this, however, is that a value of travel time (VoT) would have to be assumed, which normally differs from person to person and from time to time. In addition, a distance-dependent cost for the vehicle would have to be assigned.

We assume that the travel times are independent of the traffic load on the links. The travel time of each link is obtained by dividing the length by the free-flow speed from the volume-delay function. This approximation should be reasonable for the network we study, since the region is sparsely populated and most links have fairly low initial traffic loads. The assumption allows us to use a simpler and faster algorithm to find the fastest routes between the origins and the destinations. In principle however, there is nothing in our measures that prevents including congestion in the calculations.

4.1 **Simplifying the network**

In order to reduce the time consumption of the travel time calculations, we first simplify the network as much as possible without changing its important characteristics, even during link failures. Analysis of the network shows that for each link, there is another link by the same travel time going in the opposite direction. It is therefore possible to substitute the two directed links with one undirected link, which is assigned the travel time and the sum of the traffic loads of the old links.

As the second step we remove all centroids and connectors, since these are not part of the actual road network. Instead, we shift the travel demand to the nodes closest to the centroids (in travel time) within the detailed part of the network, which we call demand nodes. Thus, for each centroid within this region we mark its only neighbour as a demand node. For each centroid outside the region we perform a shortest path search to find the closest node within the region. During the process, we keep a record of which demand node each centroid is mapped to. Using this many-to-one map, a new $N^{\mathrm{d}} \times N^{\mathrm{d}}$ demand matrix $X = (x_{ij})$ for the demand nodes is created. Since all demand has been moved inside the study region, we can now remove the coarse parts of the network outside the region.

No shortest paths between the demand nodes will pass through “dead-end” nodes with only one neighbour, unless they are demand nodes themselves. These are therefore removed without affecting the travel times between the demand nodes. Also, non-demand “joint“ nodes with two neighbours are replaced with a direct link between the neighbours. The travel time of the new link is set to the sum of travel times of the old links, while the traffic load is preserved. Using an iterative procedure, all nodes of these two kinds are removed. The last step is to remove a few isolated nodes that are not connected to the rest of the network. After this we arrive at the final network, with N = 4,470 nodes, out of which N^{d} = 1,136 are demand nodes, and L = 6,362 undirected links, out of which L^{c} = 168 are cut links.

4.2 **Calculating the travel times**

The travel times between all demand nodes are calculated using Dijkstra’s shortest paths algorithm (Dijkstra, 1959). Starting from demand node i, we perform a shortest path search using the travel times of the links as weights. By the symmetry $t_{ij} = t_{ji}$ for all i, j, it is sufficient to calculate the travel times to the demand nodes with index $j > i$. When these have been found, the search is terminated. After repeating the search from each demand node, the travel times are stored in a symmetrical $N^{\mathrm{d}} \times N^{\mathrm{d}}$ matrix $T = (t_{ij})$. The travel time matrix is calculated first for the undamaged network and then after each link has been closed (i.e., removed), a total of $L + 1$ times.

The simplification of the network and the calculations of the travel times are performed using specially developed software (Jenelius, 2004). The all-to-all demand node shortest paths calculation for the whole network takes about 5-6 seconds on a Xeon 1.8 GHz processor, and the iterations of removing and replacing all 6,362 links take in all about 9-10 hours. These times can be expected to decrease significantly in improved future implementations, however.

5. **RESULTS**

5.1 **Link importance for the whole network**

The global importance of each link for the whole network is measured by the increase in travel time per OD pair when the link is closed (see Formula 1 in the Appendix). It can be seen from Figure 3 that the most important links are sections of the E4 European highway that stretches along the entire coast. It is clear that this road constitutes an important backbone for fast access across the network. This is to be expected since the road allows travel at high speed and since many demand nodes are located close to the coast. We can also see that the most important link, a section of the E4 located in the rocky High Coast region (indicated roughly by a frame in the figure), causes an average increase in travel time by more than ten minutes per OD pair when closed.

The demand weighted importance of each link is the increase in travel time per trip when the link is closed (Formula 2). The most important links are generally short roads within cities and do not show well on the map in Figure 4. In particular, many of the links are parts of the E4 going through Gävle and Umeå, two of the three largest municipalities in the area. These road sections therefore accommodate regional as well as much local traffic. The most important link, located in the city of Gävle to the south, causes an average increase in travel time per trip by more than one minute when closed. However, these results should be treated with some care since the network data does not include all the small streets in the cities. The effects of closing a city street may therefore appear more severe than they would be in reality.

Similarly, the links that cause the largest relative amount of unsatisfied demand when closed (Formula 3) are generally short roads in or near cities. In particular, Figure 5 shows that many of the links are located east of the E4, close to the coast where there is little room for alternative links. Others are located in the sparsely populated mountain areas and are often sections of roads that continue into Norway. Still others lie on the southern boundary and can be considered a boundary effect, since they would not divide the network if the study area was larger. When the most important link, again located in Gävle, is closed, nearly one in every forty vehicles is unable to reach its destination. Since this measure also is sensitive to inaccuracies in the network data, the results should not be taken too definitely.

5.2 **Municipality exposure to the closure of the most important link**

Global municipality exposure is measured by the average increase in travel time per origin in the municipality and destination in the whole network (Formula 4). Figure 6 shows that the municipalities in the northern parts of the area are much exposed to the closure of the most important link. Somewhat surprisingly, however, the south-east corner and Gävle in particular (indicated by an arrow in the figure), where the population and the road network are relatively dense, is the most exposed part of the area. It is in fact a single link of the E4 European highway, located in the municipality of Hudiksvall, that is the most important one for all the municipalities in this region. This is another indication of the great importance of this road.

The demand weighted exposure of a municipality is the increase in travel time per trip that begins in the municipality and ends anywhere in the network (Formula 5). Since most demand is to nearby destinations, the effects of closing a link is more local here than in the global perspective, and the most important link is always located within the same municipality. The situation is the worst in the north-western region, where the local road networks are sparser, with fewer good alternatives, than elsewhere. In Arjeplog the travellers will experience an average increase in travel time by almost one and a half hour when a certain link is closed. On the other hand, the northernmost region and a few other municipalities appear to withstand the event well. This is not the full picture, however, as we see when we turn to unsatisfied demand.

Figure 8 shows that the northernmost municipality of Kiruna is among the most exposed ones regarding the relative amount of unsatisfied demand (Formula 6). Indeed, several links in this region will cut off demand nodes from the rest of the network when closed. For the non-cut links, however, the finite increases in travel time per trip are small, as seen above. Since being unable to reach your destination is a more serious scenario, the conclusion is that unsatisfied demand should be the first measure to study. From the figure we also notice that several of the most densely populated coastal municipalities are highly exposed, while the least exposed ones are located in the centre of the region, away from the boundaries. In the municipalities of Dorotea and Sorsele, over 80 per cent of the trips will not reach their destinations if a certain link in the respective municipality is closed.

6. **CONCLUSION**

We have defined the importance of a link in a road network to be the increase in the generalised cost of travel when that link is closed. The importance can be calculated with respect to a single node, a group of nodes or the whole network by properly aggregating the travel cost. Similarly, the exposure to a certain hazardous event can be defined for a node, a group of nodes or the whole network. The exposure is then the increase in aggregated travel cost given the event. Using the unweighted travel costs gives us a perspective of global accessibility on importance and exposure, whereas weighing the travel costs with the travel demand measures importance and exposure from a perspective of economic efficiency. When the network is divided into several disconnected parts, we measure the unsatisfied demand, i.e., the travel demand between nodes separated into different parts.

We believe that these measures are intuitive and easy to aggregate at any desired level. They can also be calculated in reasonable time even for large regional networks. As an example of this, we have applied them on the network of northern Sweden. In this study we have identified the most important links for the whole network and the most exposed municipalities to a worst-case scenario event. With results such as these in hand, the implications for transport policy remain to be decided. It is obvious that a vulnerable network will lead to additional costs for individuals, companies as well as the society at large. We believe that vulnerability issues should be included in cost-benefit analyses of new road projects, which is seldom the case presently. The focus now is to a large extent on how the road network performs under normal circumstances. Hazardous events that cause road closures will inevitably occur, and when they do the situation can worsen drastically, as our study shows. Identifying and augmenting critical links can help prevent this from happening. The weakness of a link can be reduced by improving the maintenance, such as snow ploughing, and taking actions to decrease the risk of accidents. The importance of a link can be reduced by increasing the local redundancy, i.e., to provide alternative routes close at hand, such as a ferry line as an alternative to a bridge. We believe that a greater concern for issues such as these would be beneficial for the welfare of the society.

7. FURTHER RESEARCH

There are a number of possible topics for further work in this area. These include:

- Extend the study to an entire national road network such as that of Sweden. This would eliminate some boundary effects and yield more interesting results.

- Integrate the calculations with a GIS. This would allow a user to interactively study the importance of a link or the exposure of a node, municipality etc. By clicking a link, the analyst could immediately see the increase in travel cost or the amount of unsatisfied demand that a shutdown of that link would cause, both for the network as a whole and for the most affected regions. In a corresponding way, selecting a node or a region could inform the user which links it is most dependent upon. Calculating the measures on demand, however, requires a highly efficient travel cost algorithm.

- Speed up the shortest path and travel cost calculations. This will be necessary if the measures are to be calculated for very large networks in reasonable time. An algorithm especially suited for the particular network should be identified and used (see Zhan and Noon, 1998). Some promising work based on re-optimisation has also been done (Holmgren, 2004b).

- Include congestion and elastic demand in the calculations. When the network contains relatively densely populated areas such as southern Sweden, this would arguably yield a more realistic transport model. However, including congestion would probably also increase the time consumption of the calculations significantly, in conflict with the previous points.

- Study how to best reduce exposure and importance by adding new links. Given a certain budget, how are new links to be drawn in order to minimize the exposure of a region to a certain event? This is a variant of the discrete network design problem, which is recognized as a challenging optimisation problem (see Bell and Iida, 1997).

- Develop the probability part of vulnerability and criticality. An important issue is how to model the weakness of each link, i.e., how to identify the properties that make it likely to fail. With detailed information about previous incidents, this should be possible to answer by using a regression analysis approach. There is also a need for more realistic models and scenario building of the hazardous events that can damage the network. Again, data from real disturbances in the road network can be used for this task.

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APPENDIX: FORMULAS

For definitions of the terms, see section 2.2.

Link importance for the whole network

- Global importance: Increase in travel time per OD pair

$$\frac{1}{N^{\mathrm{d}}(N^{\mathrm{d}}-1)}\sum_{i}\sum_{j\neq i}(t_{ij}^{(k)}-t_{ij}^{(0)}),\quad k\in E^{\mathrm{nc}} \tag{1}$$

- Demand weighted importance: Increase in travel time per trip

$$\frac{\sum_{i}\sum_{j\neq i}x_{ij}(t_{ij}^{(k)}-t_{ij}^{(0)})}{\sum_{i}\sum_{j\neq i}x_{ij}},\quad k\in E^{\mathrm{nc}} \tag{2}$$

- Relative unsatisfied demand

$$\frac{\sum_{i}\sum_{j\neq i}u_{ij}^{(k)}}{\sum_{i}\sum_{j\neq i}x_{ij}},\quad k\in E \tag{3}$$

Municipality exposure to the closure of the most important link

- Global exposure: Increase in travel time per OD pair

$$\max_{k\in E^{\mathrm{nc}}}\frac{1}{N_m^{\mathrm{d}}(N^{\mathrm{d}}-1)}\sum_{i\in V_m^{\mathrm{d}}}\sum_{j\neq i}(t_{ij}^{(k)}-t_{ij}^{(0)}) \tag{4}$$

N_m^{d} is the number of demand nodes in municipality *m*.

- Demand weighted exposure: Increase in travel time per trip

$$\max_{k\in E^{\mathrm{nc}}}\frac{\sum_{i\in V_m^{\mathrm{d}}}\sum_{j\neq i}x_{ij}(t_{ij}^{(k)}-t_{ij}^{(0)})}{\sum_{i\in V_m^{\mathrm{d}}}\sum_{j\neq i}x_{ij}} \tag{5}$$

- Relative unsatisfied demand

$$\max_{k\in E}\frac{\sum_{i\in V_m^{\mathrm{d}}}\sum_{j\neq i}u_{ij}^{(k)}}{\sum_{i\in V_m^{\mathrm{d}}}\sum_{j\neq i}x_{ij}} \tag{6}$$

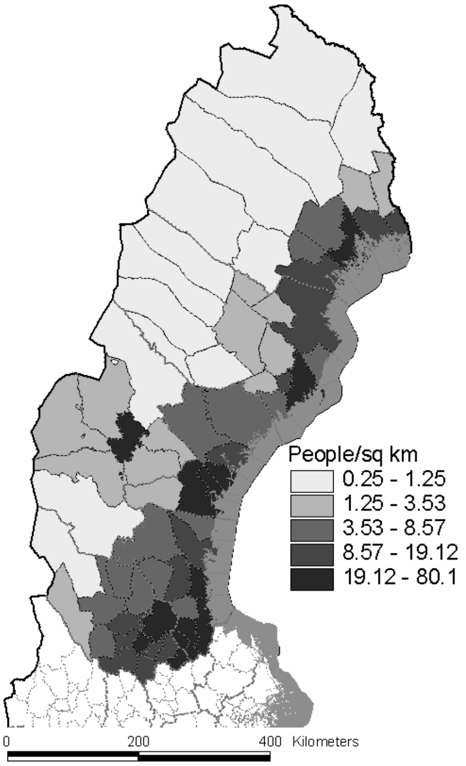

Figure 1: Population density of the municipalities of northern Sweden, as of September 30, 2004. From Statistics Sweden and National Atlas of Sweden.

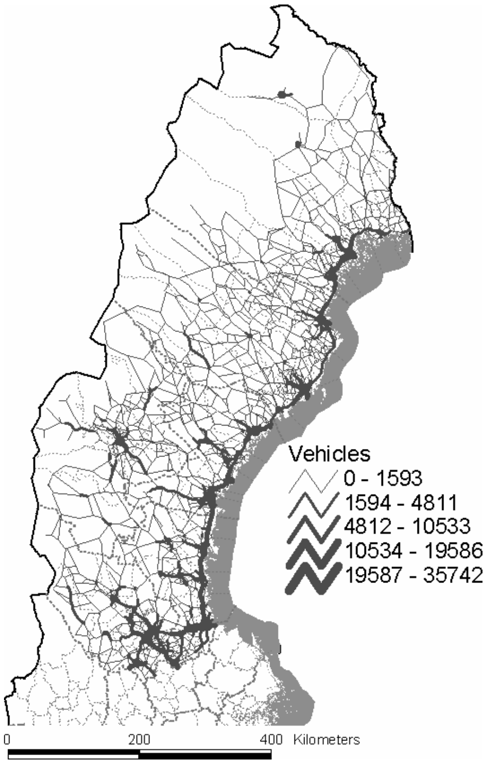

Figure 2: Traffic load on the links in the simplified road network, from an assignment by SRA.

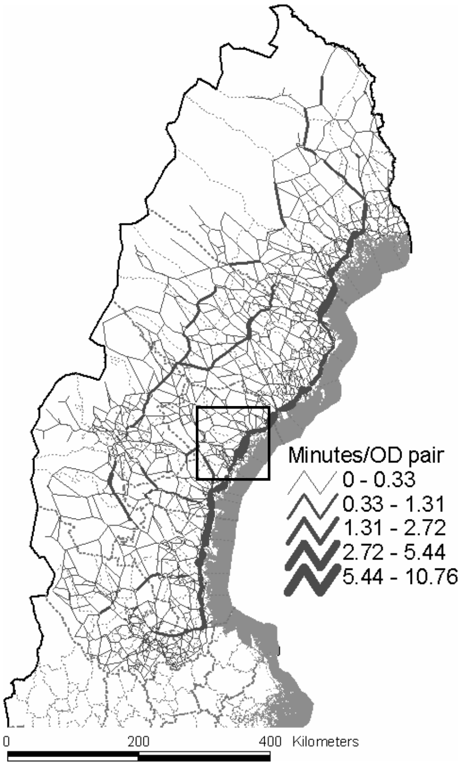

Figure 3: Global link importance for the whole network (Formula 1).

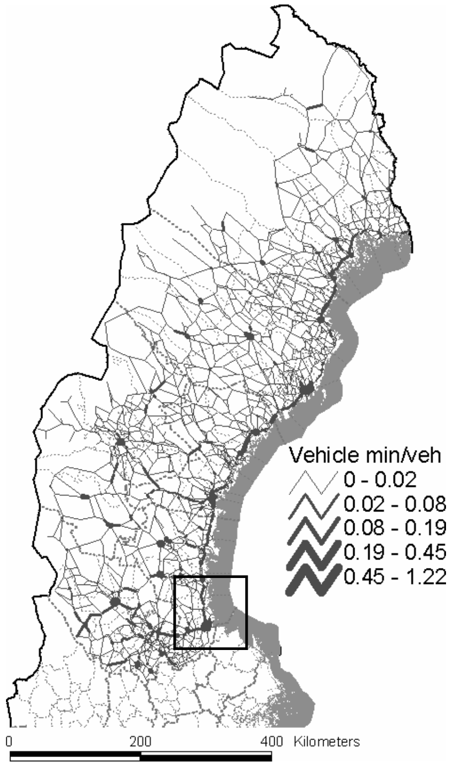

Figure 4: Demand weighted link importance for the whole network (Formula 2).

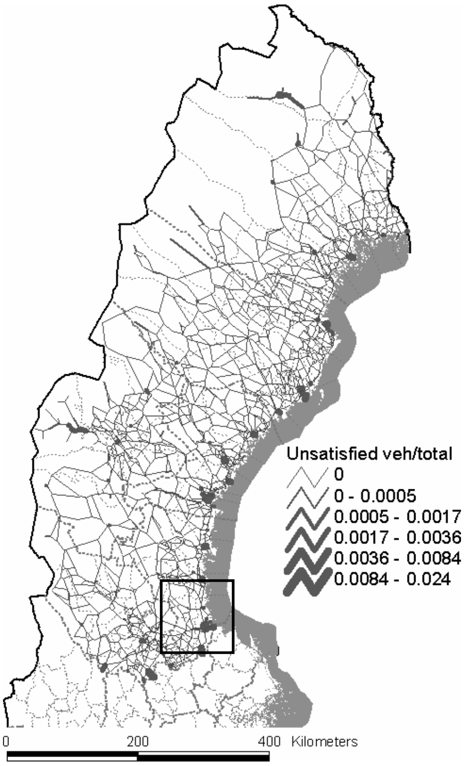

Figure 5: Unsatisfied demand related link importance for the whole network (Formula 3).

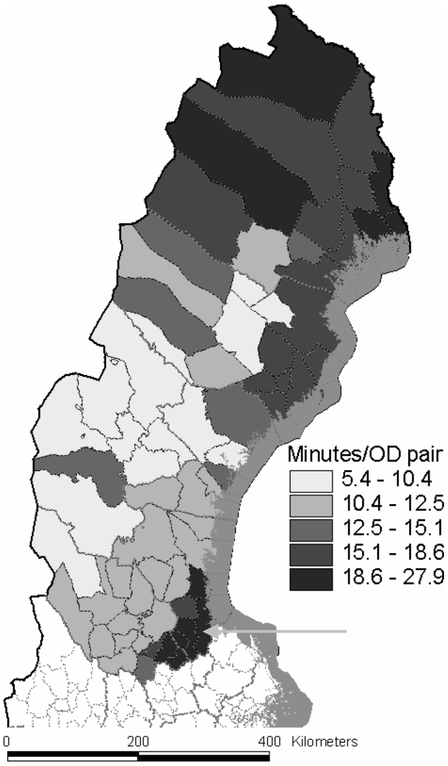

Figure 6: Global municipality exposure to the closure of the most important link (Formula 4).

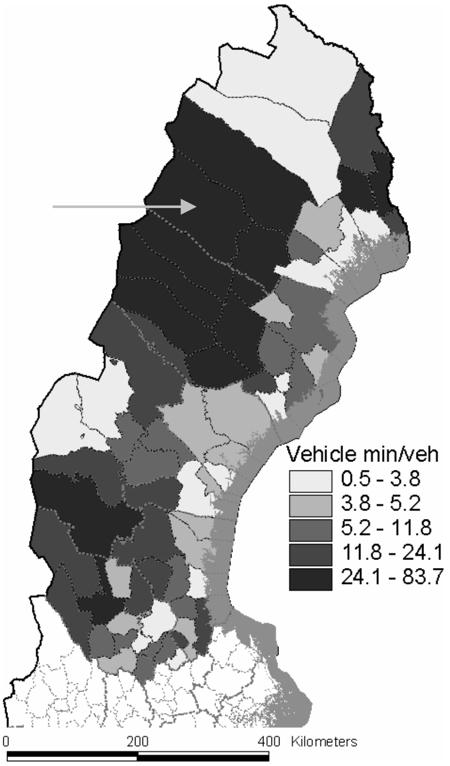

Figure 7: Demand weighted municipality exposure to the closure of the most important link (Formula 5).

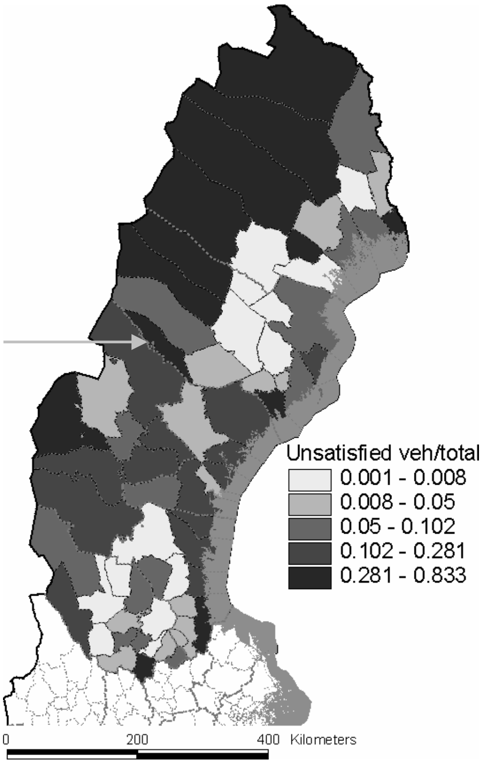

Figure 8: Unsatisfied demand related municipality exposure to the closure of the most important link (Formula 6).